

# Scales, quantity & degree

## **Lecture 1: Quantifiers**

Rick Nouwen (Utrecht)

October 2013

# Scales, quantity and degree

$$S = \langle X, > \rangle \quad \text{or} \quad S = \langle X, \models \rangle$$

- 1 Quantifiers
- 2 Modified numerals
- 3 Intensifiers

# What is a quantifier?

Every  
Some  
Several  
A few

Natural language determiners

Every student  
Some rabbits  
Several parts that move

Determiner phrases

$\forall$     $\exists$

Predicate logical syncategorema

$\lambda X.\lambda Y.|X \cap Y| > 3$

Generalised quantifiers

# What is a quantifier?

- articles: **a, the**, ...
- determiners: **every, most**, ...
- number words: **one, two, three**, ...
- comparatives: **fewer than five hundred, more than just a few**, ...
- superlatives: **at most five, at least twelve**, ...
- PPs: **between sixty and seventy, up to two hundred**, ...
- adjectives: **(very) many, (too) few**, ...
- modifications: **almost every, exactly five**, ...
- coordinations: **most but not all, two or three**, ...

\_\_\_\_\_ **monkey(s) is/are asleep in the zoo**

'I see nobody on the road,' said Alice.

'I only wish *I* had such eyes,' the King remarked in a fretful tone.  
'To be able to see Nobody! And at that distance, too!

Lewis Carrol, Through the looking glass, and what Alice found there

According to the story, there was a man with a headache, who saw the advertisement:

nothing acts faster than Aspirin™

— so at once he went and took nothing.

*(after Wilfrid Hodges, Logic, 1977)*

According to the story, there was a man with a headache, who saw the advertisement:

nothing acts faster than Aspirin™

— so at once he went and took nothing.

*(after Wilfrid Hodges, Logic, 1977)*

— so at once he went and took Aspirin

# Plan

- Quantifiers are obviously not referring terms

quantifiers ~ quantities

- Option 1: quantifiers stand proxy for quantities
- Option 2: quantifiers express relations between quantities  
*generalised quantifier theory*
- In favour of a much richer theory of quantifier meaning



- 1 Words & numbers
- 2 Generalised Quantifiers
- 3 Logical Quantifiers
- 4 Complication 1: Many
- 5 Complication 2: Perspective
- 6 The syntactic force of generalised quantifiers
- 7 Towards decomposition: modified numerals

- 1 Words & numbers
- 2 Generalised Quantifiers
- 3 Logical Quantifiers
- 4 Complication 1: Many
- 5 Complication 2: Perspective
- 6 The syntactic force of generalised quantifiers
- 7 Towards decomposition: modified numerals

# The psychology of words versus numbers

Bryant & Norman 1980; Beyth & Marom 1982; Wallsten et.al. 1986; Erev & Cohen 1990; Renooij & Witteman 1999

- main application
  - expert knowledge communication
  - communication guidelines (e.g. medical professions)
- does/should an expert use words or numbers?
- how do words/phrases correspond with explicit cardinality or frequency
- this type of research presupposes that natural language quantifiers express quantities

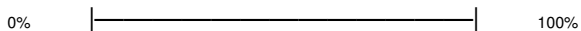
# The psychology of words versus numbers

Bryant & Norman 1980; Beyth & Marom 1982; Wallsten et.al. 1986; Erev & Cohen 1990; Renooij & Witteman 1999

- Hearers prefer numbers
- Speakers prefer words

# The psychology of words versus numbers

Bryant & Norman 1980; Beyth & Marom 1982; Wallsten et.al. 1986; Erev & Cohen 1990; Renooij & Witteman 1999



*many, quite a few, few, a few, very  
few, a lot, not many, several*

# The psychology of words versus numbers

Bryant & Norman 1980; Beyth & Marom 1982; Wallsten et.al. 1986; Erev & Cohen 1990; Renooij & Witteman 1999

- Huge between-subject variation
- Considerable overlap between words

# Against quantifiers as words for amounts

Linda Moxey, Anthony Sanford (Glasgow)

- Too many quantifiers (in a single free production experiment Moxey (1986) observed 182 different quantity expressions)
- Moxey & Sanford 1993
  - each subject can assign just one number to one quantifier on one occasion only (450 subjects)
  - no context effects, no comparison among quantifiers
  - results: impossible to distinguish between *a few*, *only a few*, *not many*, *few* and *very few*
- Support comes from studies on intensifiers such as *very* (Wright et.al. 1995)

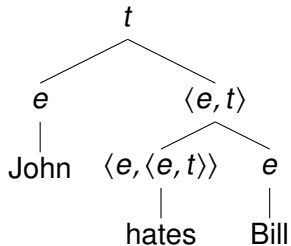
# Words and numbers

- Quantifiers do not go proxy for numbers
- In linguistic semantics the relation between quantifiers and quantities is a bit more complex, though

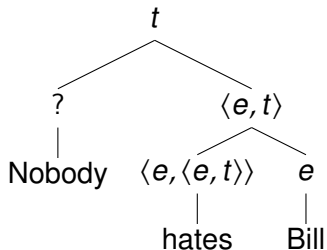
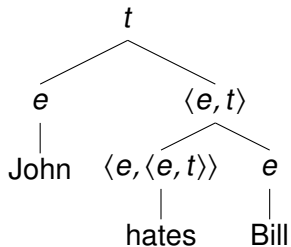


- 1 Words & numbers
- 2 Generalised Quantifiers**
- 3 Logical Quantifiers
- 4 Complication 1: Many
- 5 Complication 2: Perspective
- 6 The syntactic force of generalised quantifiers
- 7 Towards decomposition: modified numerals

# A dilemma for compositionality



# A dilemma for compositionality



# Quantifiers

## Aristotelian syllogisms

All mortals die.

Some men are mortal.

---

Some men die.

## Medieval studies on quantification

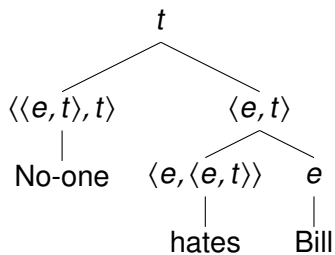
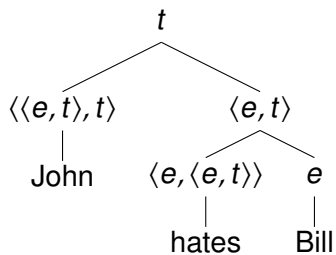
All men are mortal

*subject* *predicate*

Question: What does the subject express?

# The modern solution

Montague, Barwise & Cooper, Keenan & Stavi



## Quantifiers as sets of sets

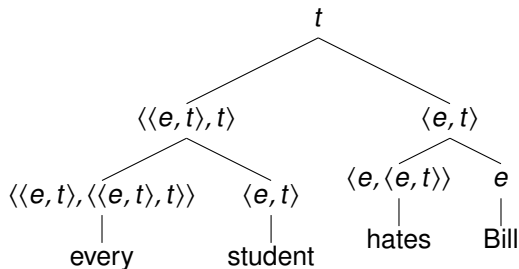
Let  $P$  be the set of people in the domain.

$$\llbracket \text{everyone} \rrbracket = \lambda X. P \subseteq X$$

$$\llbracket \text{someone} \rrbracket = \lambda X. P \cap X \neq \emptyset$$

$$\llbracket \text{noone} \rrbracket = \lambda X. P \cap X = \emptyset$$

# Determiners as relations between sets



$\llbracket \text{every} \rrbracket = \lambda X. \lambda Y. X \subseteq Y$

$\llbracket \text{some} \rrbracket = \lambda X. \lambda Y. X \cap Y \neq \emptyset$

$\llbracket \text{no} \rrbracket = \lambda X. \lambda Y. X \cap Y = \emptyset$

# Generalised Quantifier Theory

Barwise & Cooper, Keenan, van Benthem, Westerstahl

- a collection of linguistic and mathematical insights
  - properties of linguistic and mathematical quantifiers
  - linguistic universals concerning such properties
  - main focus is on *generalisations*
- 
- to a much lesser extent: processing aspects of quantifiers
  - to a much lesser extent: linguistic properties of particular quantifiers



# Example: the definiteness effect

Milsark 1977, Barwise & Cooper 1981, Keenan 1987

## (1) Weak quantifiers

- a. There are at least three gnomes in the garden.
- b. There are some biscuits left in the fridge.
- c. There are no aliens on mars.
- d. There are fifty-two typos in the manuscript.

## (2) Strong quantifiers

- a. \*There is every student in the classroom.
- b. \*There are most biscuits on this plate.
- c. \*There are less than half the gnomes in the garden.
- d. \*There are not all aliens on mars.

The weak/strong distinction can be made formally explicit in terms of formal properties of quantifiers

# Strong versus weak

**Symmetry:**  $Q(X)(Y) \leftrightarrow Q(Y)(X)$

Strong quantifiers are not symmetrical:

If every student is a spy, then every spy is a student

If most students are spies, then most spies are students

Weak quantifiers are symmetrical:

If some students are spies, then some spies are students

If no students are spies, then no spies are students

Existential-there sentences only admit symmetrical quantifiers

- 1 Words & numbers
- 2 Generalised Quantifiers
- 3 Logical Quantifiers**
- 4 Complication 1: Many
- 5 Complication 2: Perspective
- 6 The syntactic force of generalised quantifiers
- 7 Towards decomposition: modified numerals

# Generalised Quantifier Theory

----- is/are asleep in the zoo

Assumption of homogeneity within the class of GQs

# Isomorphism invariance

For any  $U$  and  $U'$ ,

if  $\pi : U \rightarrow U'$  is a bijection, then

$$Q_U(X, Y) \rightarrow Q_{U'}(\pi(X))(\pi(Y))$$

# Isomorphism invariance

$$\xrightarrow{\pi}$$

$U$	$U'$
a	b
b	c
c	e
d	a

$$X = \{a, c, d\}$$

$$Y = \{a\}$$

$$Q = \lambda A. \lambda B. |A \cap B| < 10$$

$$\begin{aligned} Q(X)(Y) &\leftrightarrow \\ |\{a, c, d\} \cap \{a\}| < 10 &\leftrightarrow \\ |\{a\}| < 10 \end{aligned}$$

$\Downarrow_{pi}$

$$\begin{aligned} Q(\pi(X))(\pi(Y)) &\leftrightarrow \\ |\{b, e, a\} \cap \{b\}| < 10 &\leftrightarrow \\ |\{b\}| < 10 \end{aligned}$$

# Logicity

- Logicity is the notion that purely logical operators are not about particular entities but are *topic neutral*
- van Benthem: *Quantifiers* are expressions that satisfy *isomorphism invariance*
- Most of the mathematical work on GQs concerns such logical quantifiers
  
- $\llbracket \text{John} \rrbracket = \lambda A.A(j)$
- $\llbracket \text{Every ... but John} \rrbracket = \lambda A.\lambda B.(B \cup \{j\}) \subseteq A$
  
- With some extra assumptions: logical quantifiers are those that rely solely on cardinality

# Logicality: the tree of numbers

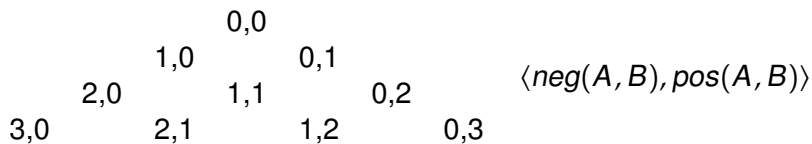
For a pair  $A$  and  $B$ , let  $pos(A, B)$  be  $|A \cap B|$  and  $neg(A, B)$  be  $|A \setminus B|$

$$\llbracket \text{every} \rrbracket(A)(B) \leftrightarrow neg(A, B) = 0$$

$$\llbracket \text{no} \rrbracket(A)(B) \leftrightarrow pos(A, B) = 0$$

$$\llbracket \text{most} \rrbracket(A)(B) \leftarrow pos(A, B) > neg(A, B)$$

$$\llbracket \text{more than 2} \rrbracket(A)(B) \leftrightarrow pos(A, B) > 2$$





# Generalised Quantifier Theory

- Logicality represents extreme view within GQT
- There exists a subclass of logical quantifiers
- These only express a relation between their arguments and a
  
- Limited applicability to natural language
- There is more to quantifiers than cardinality
- Even the purported logical ones

- 1 Words & numbers
- 2 Generalised Quantifiers
- 3 Logical Quantifiers
- 4 Complication 1: Many**
- 5 Complication 2: Perspective
- 6 The syntactic force of generalised quantifiers
- 7 Towards decomposition: modified numerals

## Many is non-extensional

- Logicality assumes that at the heart of quantifier semantics is cardinality comparison
- The arguments of a quantifier are taken for granted
- $[[\text{many}]] = \lambda A.\lambda B.|A \cap B| > m$ 
  - Many lawyer attended the meeting this year.
  - Many women attended the meeting this year.

## Many is non-extensional

- Logicality assumes that at the heart of quantifier semantics is cardinality comparison
- The arguments of a quantifier are taken for granted
- $[[\text{many}]] = \lambda A.\lambda B.|A \cap B| > m$ 
  - Many lawyer attended the meeting this year.
  - Many women attended the meeting this year.

Imagine a conference of lawyers and policemen where normally 60 lawyers and 40 policemen attend. Also, on average, only 10 attendants are women. This year, there are only 20 lawyers, but a staggering 80 policemen. Strikingly, all the lawyers happen to be women and all the policemen are men.

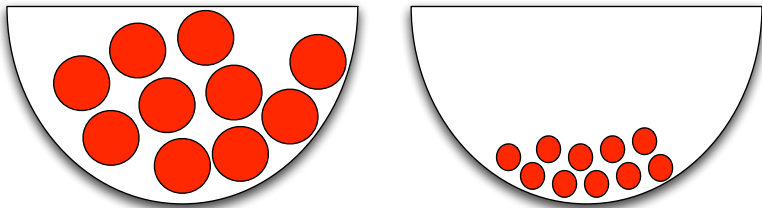
## Many is non-extensional

$$\llbracket \text{many} \rrbracket = \lambda A_{\langle s, \langle e, t \rangle \rangle} \cdot \lambda B_{\langle s, \langle e, t \rangle \rangle} \cdot$$

$$\forall w \in \text{CONB} : |A(w^*) \cap B(w^*)| > |A(w) \cap B(w)|$$

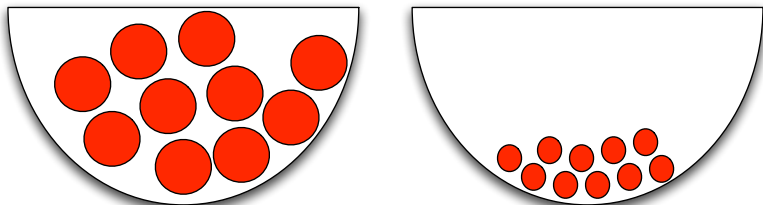
Shalom Lappin, 2000

# Newstead & Coventry 2000



*There are many marbles in the bowl*

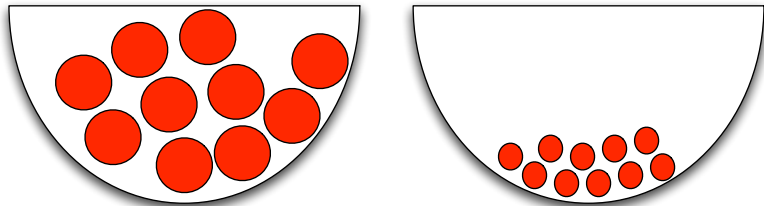
## Newstead & Coventry 2000



*There are many marbles in the bowl*

“The studies all used a task in which participants rated the appropriateness of quantifiers describing the number of balls in a bowl. The size of the balls was found to have an effect: Identical numbers of balls were given different ratings depending on ball size.” Newstead & Coventry 2000

# Newstead & Coventry 2000



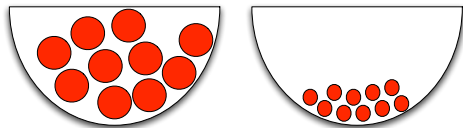
*There are many marbles in the bowl*

“The results are interpreted as indicating that quantifiers carry little specific meaning in themselves but instead derive their meaning from the context in which they occur.” Newstead & Coventry 2000



# The vagueness of *many*

- The conclusion of Newstead & Coventry becomes trivial if we do not think of *many* as a quantifier
- but if we instead think of it as a relative adjective
- i.e. what N&C uncovered is simply standard context-dependence and vagueness
- the effects are clearly limited to a subset of the set of GQs



*There are more than seven marbles in the bowl*

## Many as a relative adjective

John is a tall basketball player

John is a tall toddler

There are many ants in my back garden

There are many mole hills in my back garden

# Vagueness

A predicate is vague

- If it has borderline cases (and if there are cases that are borderline borderline, etc.)
- If it can be part of a sorites paradox

# Vagueness in relative adjective

- Key semantic contrast:

- the positive form: John is tall

- the comparative form: John is taller than Bill

vague

crisp

# Comparison and Vagueness

Kennedy 2007

Uranus is big, compared to Venus.

Uranus is bigger than Venus.

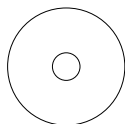


Figure 1: Uranus (51,118 km) vs. Venus (12,100 km)

Uranus is bigger than Neptune.

#Uranus is big, compared to Neptune.

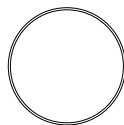


Figure 2: Uranus (51,118 km) vs. Neptune (49,500)

## Vagueness with **Many**

There are many mole hills in my garden

There are more mole hills in my garden than in yours

# Many as a relative adjective

cf. Solt 2006, 2007

- Distribution of **very**: modifies adjectives
  - I am very (\*much) tall
  - I like you very \*(much)
  - I am very \*(much) into heavy metal music
- I found very many (\*much) mistakes
  
- His good qualities are many
  
- The flaws in the proposal were many and serious

# Vagueness and Informativity

To use words specifically you also have to avoid vague terms like "many," "few," and "difficult." In their place you should use precise words like "four" or "illegal." What does many mean? It means different things to different people. But four (whether it is four or four million) is a measurable amount. If you wanted to refer to how many widgets your company had you might be tempted to reply "a lot" if you found that your company had an entire warehouse full of them. However, your boss might respond "that's not even a year's worth." [..] The words "many," "large," and "important" mean something different to each of us. As a writer you should constantly strive to select simple, straightforward words that mean the same thing to most people.

(University of Florida Precize Writing Guide)



# Vagueness and informativity

Be specific with numbers and avoid vague terms like **many**, **a lot**, and **most**.

(Associated Press Stylebook and Libel Manual)

<http://academics.smcvt.edu/dlynch/apstyle.htm>

# How to be precise

- Message: vagueness (and imprecision) leads to un informativity
  - (i) There were exactly 34 people at my party last week.
  - (ii) There were many people at my party last week.
  - (iii) There were more than 10 people at my party last week.
- All three sentences are about how many people attended my party. But only (i) gives a precise answer.

# How to be precise

- Message: vagueness (and imprecision) leads to un informativity
  - (i) There were exactly 34 people at my party last week.
  - (ii) There were many people at my party last week.
  - (iii) There were more than 10 people at my party last week.
- All three sentences are about how many people attended my party. But only (i) gives a precise answer.
- All three can give informative answers:
  - (i): the number of guests was 34
  - (ii): the number of guests was satisfactory
  - (iii): the number of guests was sufficient

- 1 Words & numbers
- 2 Generalised Quantifiers
- 3 Logical Quantifiers
- 4 Complication 1: Many
- 5 Complication 2: Perspective**
- 6 The syntactic force of generalised quantifiers
- 7 Towards decomposition: modified numerals

# Moxey and Sanford's perspective approach

Moxey & Sanford 2000

- Logicality has it that quantifiers express cardinality relations
- Moxey and Sanford: Quantifiers express *a perspective on a quantity*
- Rather than providing a quantity
- they *describe* it from a certain perspective

# Moxey and Sanford's perspective approach

Sanford et al. 2002

In the train disaster, a few people were seriously injured, which is a #good/bad thing.

In the train disaster, few people were seriously injured, which is a good/#bad thing.

# Moxey and Sanford's perspective approach

Sanford et al. 2002

Thankfully, not quite all passengers died in the crash.

#Thankfully, almost all passengers died in the crash.

Thankfully, few passengers died in the crash.

#Thankfully, a few passengers died in the crash.

# Perspective

Moxey and Sanford 1993, Nouwen 2003 — complement anaphora

Nearly all of the fans went to the match.  
They cheered their team on at every opportunity.

Not quite all of the fans went to the match.  
They watched it at home on TV instead.



# Perspective

Horn 2002

Few students got this question right. For example, Bill didn't /  
?Bill did.

A few students got this question right. For example, Bill did /  
???Bill didn't

## Perspective effects and GQT

A set of sets  $Q$  is  $\text{MON}\uparrow$  iff  $Q(X) \wedge X \subseteq X' \Rightarrow Q(X')$

A set of sets  $Q$  is  $\text{MON}\downarrow$  iff  $Q(X) \wedge X' \subseteq X \Rightarrow Q(X')$

a few passengers is  $\text{MON}\uparrow$

few passengers is  $\text{MON}\downarrow$

# Perspective and monotonicity

Nouwen 2003

Most students went to the party.  
They had a lot of fun.

Most students went to the party.  
#They were too busy.

Very few of the students went to the party.  
They (still) had a lot of fun.

Very few of the students went to the party.  
They were too busy.

However. . .

# The limits of monotonicity

Thankfully, not quite all politicians are corrupt

#Thankfully, almost all politicians are corrupt

# The limits of monotonicity

Thankfully, not quite all politicians are corrupt

#Thankfully, almost all politicians are corrupt

You have to answer almost all questions correctly if you want to pass

#You have to answer not quite all questions correctly if you want to pass

almost X entails but does not assert  $\neg X$

not quite X entails and asserts  $\neg X$

# The limits of monotonicity

SALE! Up to 60% reduction!

#SALE! At most 60% reduction!

# Interim summary

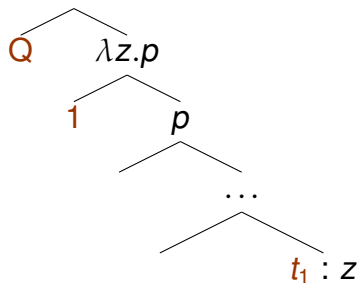
- There is more to quantifiers than cardinality
- or cardinality comparison
  - intensionality
  - vagueness and context-dependence
  - perspective
  
- There is no *one* semantic template for quantity expressions
  
- Next: some more complications
- But: the tools of GQT are indirectly relevant

- 1 Words & numbers
- 2 Generalised Quantifiers
- 3 Logical Quantifiers
- 4 Complication 1: Many
- 5 Complication 2: Perspective
- 6 The syntactic force of generalised quantifiers**
- 7 Towards decomposition: modified numerals

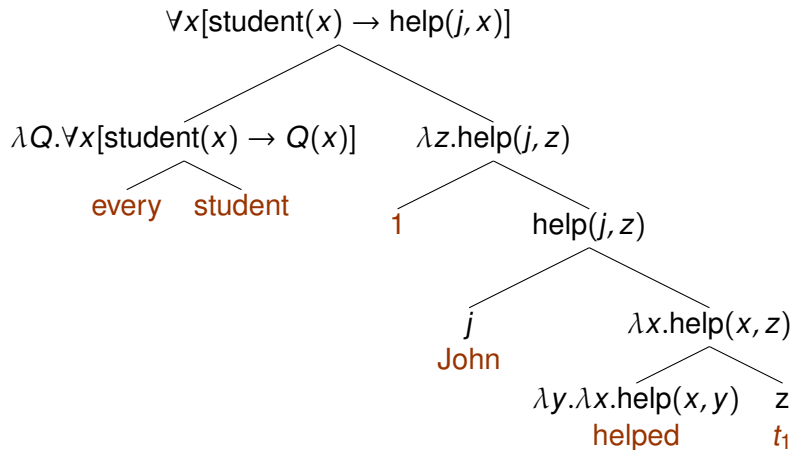


# The syntactic force of $\langle\langle e, t \rangle, t\rangle$

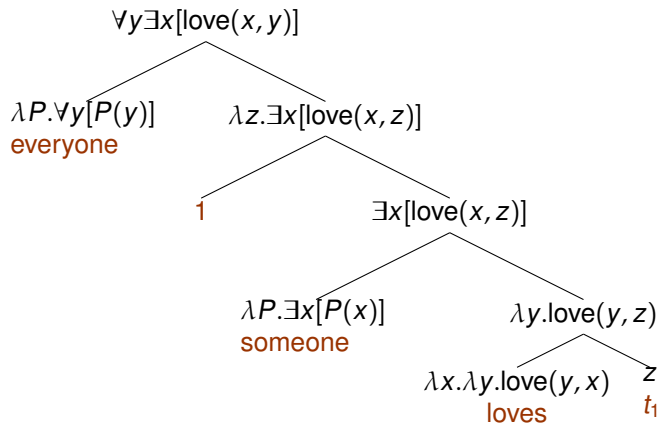
- Quantifiers (type  $\langle\langle e, t \rangle, t\rangle$ ) may move and adjoin at a higher node (of type  $t$ )
- leaving behind a trace of type  $e$



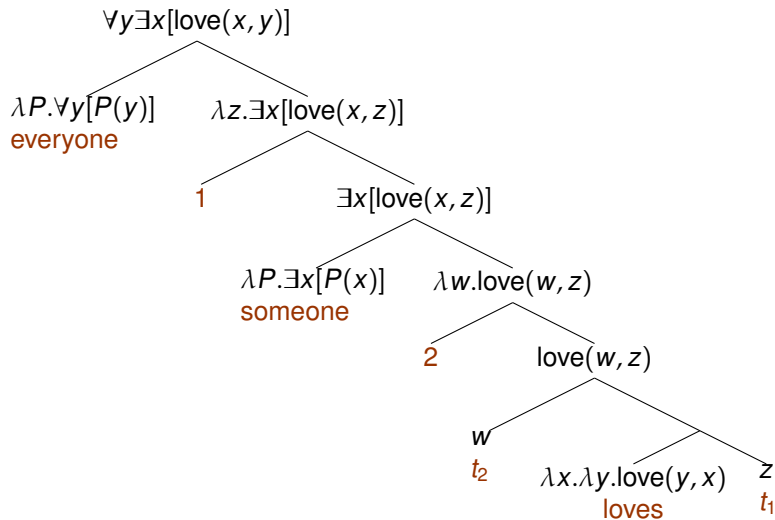
# Quantifier raising



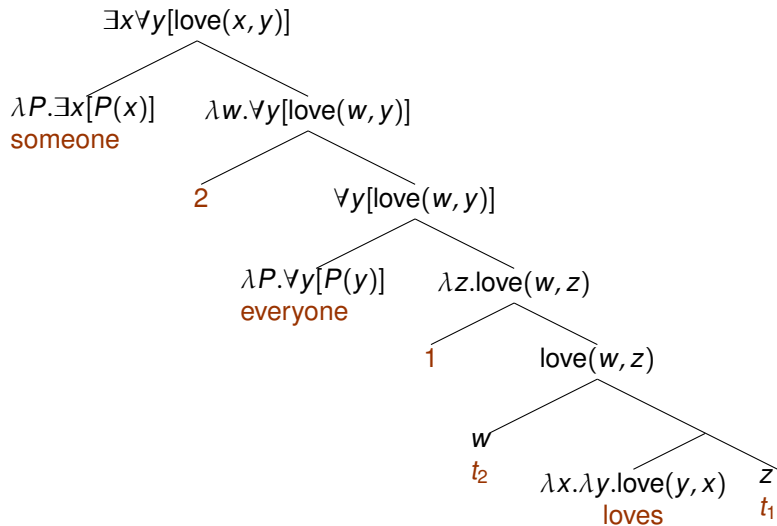
# Someone loves everyone



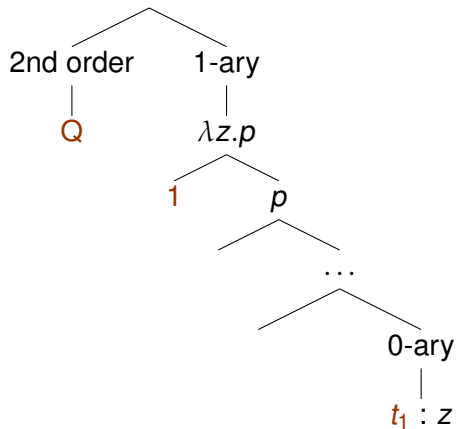
# Someone loves everyone



# Someone loves everyone



# The versatility of QR



# The versatility of QR

- Operators that raise are of type  $\langle\langle\alpha, t\rangle, t\rangle$
  - There is  $\alpha$ -lambda abstraction over the landing site's sister
  - Traces are of a simplex type  $(\alpha)$
- 
- Example: the comparative

# DegP movement

Heim 2000

- What does the comparative express?
- First attempt:  $\llbracket \text{taller} \rrbracket = \lambda x. \lambda y. y\text{'s height} > x\text{'s height}$
- **John is taller than Bill** is true iff John's height exceeds Bill's
- **John is taller than 6'**
- **The table is longer than the room is wide**
- Second attempt:  $\llbracket \text{taller} \rrbracket = \lambda d. \lambda y. y\text{'s height} > d$



# The comparative

Heim 2000

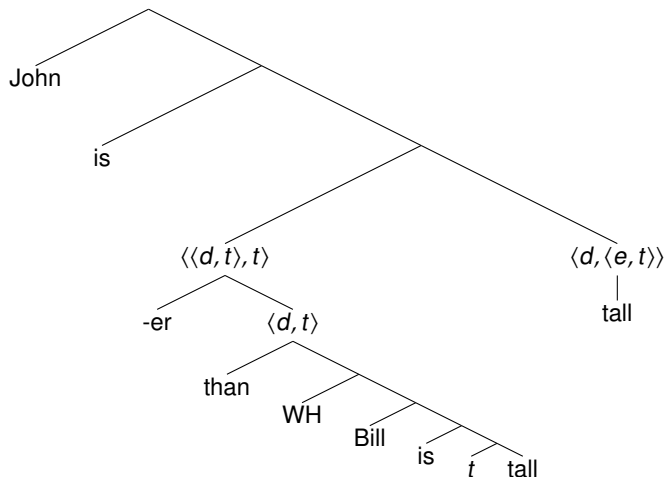
- $\lambda d.\lambda y.y$ 's height  $> d$
- How does a **than**-clause denote a degree? (It doesn't)
- **than** WH<sub>*i*</sub> Bill is *t<sub>i</sub>* tall
- \*John is healthier than Mary wants to do fitness in order to be
- How healthy does Mary want to do fitness in order to be?
- $\lambda d$ .Bill is *d* tall

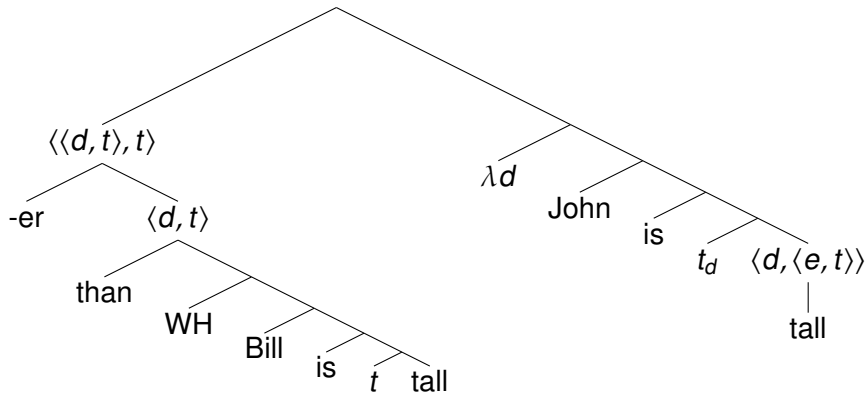
Consequence:  $\llbracket \text{-er} \rrbracket = \lambda D_{\langle d,t \rangle}.\lambda y.y$ 's height  $> \max(D)$

# Comparative morphology as a generalised quantifier

Heim 2000

$$\llbracket \text{-er} \rrbracket = \lambda D_{\langle d,t \rangle} . \lambda D'_{\langle d,t \rangle} . \max(D') > \max(D)$$





# The comparative

Heim 2000

(This draft is 10 pages.)

The paper is required to be exactly 5 pages longer than that.

- $\square[\max(\lambda d.\text{long}(p, d))] = 15pp$   
the paper should be 15 pages long
- $\max(\lambda.\square\text{long}(p, d)) = 15pp$   
the minimum number of pages that is acceptable for the paper is 15

(This draft is 10 pages.)

The paper is allowed to be exactly 5 pages longer than that.

- $\diamond[\max(\lambda d.\text{long}(p, d))] = 15pp$   
it's okay if the paper turns out 15 pages long
- $\max(\lambda.\diamond\text{long}(p, d)) = 15pp$   
the upper page limit is 15 pages

# The comparative

Heim 2000

(This draft is 10 pages.)

The paper is required to be less long than that.

- the paper should be shorter
- the minimum number of pages that is acceptable for the paper  $< 10$

The second reading might be difficult to get, but it is available. Try:

(For the Nigella Lawson version of this cake I used 6 sticks of butter.) The Delia Smith version requires less butter than that.

# The comparative

Heim 2000

Not all intensional verbs behave similarly. Heim's examples:

The paper should be less long than that.

- #It's not required for it to be as long as that

The paper is supposed to be less long than that.

- #It's not required for it to be as long as that

I want the paper to be less long than that.

- #I don't require it to be as long as that

(My prediction: Bill will break the world record long jump (8m95cm). It turned out he only jumped 8m80cm.) I predicted Bill to jump exactly 15cm further than that.

# New interim summary

- There is more to quantifiers than quantity
- Some quantity expressions simply are not quantifiers
- Some quantifiers are not quantity expressions

many

almost all / not quite all

-er

- There is no homogeneous class of *quantifiers*, unless we focus on very narrow properties
  - syntactic mobility
  - logicity?

- 1 Words & numbers
- 2 Generalised Quantifiers
- 3 Logical Quantifiers
- 4 Complication 1: Many
- 5 Complication 2: Perspective
- 6 The syntactic force of generalised quantifiers
- 7 Towards decomposition: modified numerals**



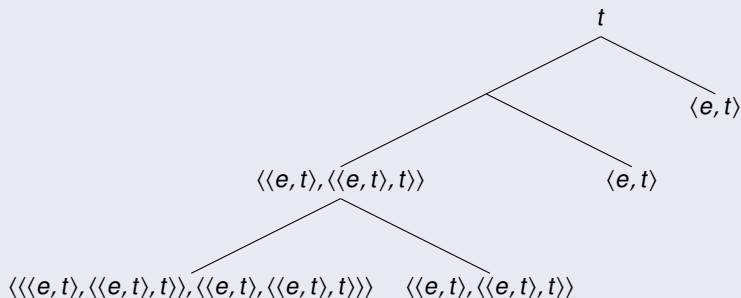
# Two issues

## Three supposedly equivalent determiners

more than two  
at least three  
three

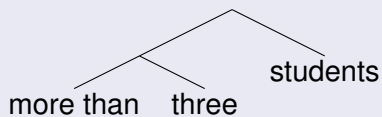
$$\lambda X.\lambda Y.|X \cap Y| \geq 3$$

## A universal structure for quantified expressions

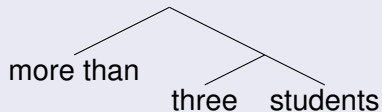


# The structure of modified numerals

GQT



An alternative



# The structure of modified numerals: Hackl 2001

- (i) #More than one student is meeting
- (ii) At least two students are meeting
  
- (iii) #More than 9 people got married on Saturday
- (iv) At least 10 people got married on Saturday
  
- (v) #John separated more than one animal
- (vi) John separated at least two animals

Hackl's proposal: [ more than [ two NP VP ] ]

# The structure of modified numerals: Focus-sensitivity

Krifka 1999, Geurts & Nouwen 2007

At least THREE boys left. (Maybe four)

At least three BOYS left. (Maybe some girls too)

At least it isn't RAINING. (Maybe the sun will even shine)

This behaviour is unexpected if **at least** modifies **three**.

# Modified numerals show signs of DegP movement

Hackl 2000

Bill needs to score fewer than 10 points to win.

- Bill will win only if he doesn't score 10 or more points (available, but unlikely)
- The minimum number of points Bill needs to score to win is 9 or fewer. (available)

Bill is allowed to eat fewer than 10 cookies.

- It's okay if Bill eats 9 or fewer cookies. (available)
- Bill shouldn't eat more than 9 cookies. (available)

# Hackl's comparative semantics

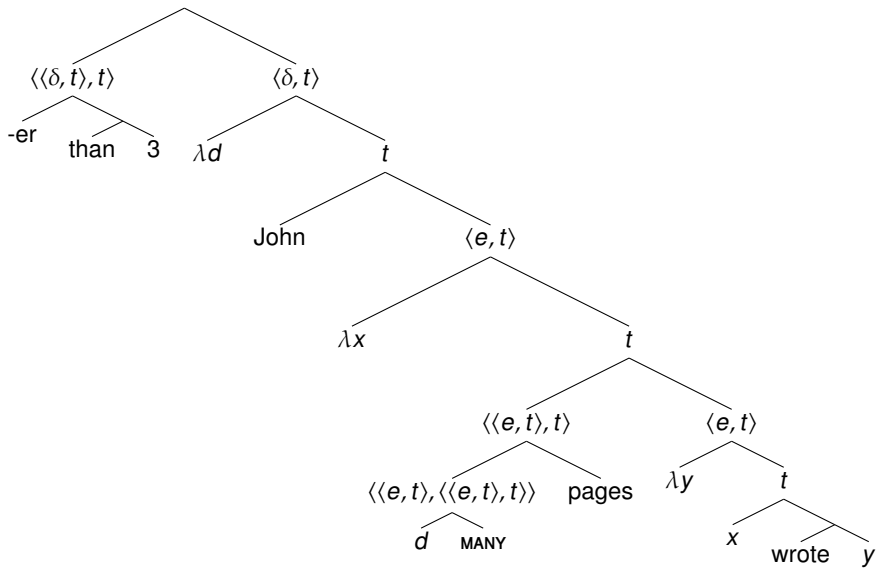
Hackl 2000

$$\llbracket \text{MANY} \rrbracket = \lambda d. \lambda X. \lambda Y. \exists x [\#x = d \wedge X(x) \wedge Y(x)]$$

$$\llbracket 3 \rrbracket = 3$$

$$3 \text{ pages} \rightsquigarrow \llbracket [3 \text{ MANY}] \text{ pages} \rrbracket \rightsquigarrow \lambda Y. \exists x [\text{page}(x) \wedge Y(x) \wedge \#x = 3]$$

John wrote more than three pages.







# Hackl's comparative semantics

Hackl 2000

[ [fewer than 10 ] [  $\lambda d$  [ allowed [ [  $d$  many ] cookies ] [  $\lambda x$  [ Bill eat  $x$  ] ] ] ] ] ]

Sentence:  $\max(\lambda d. \diamond \exists x [\text{cookies}(x) \wedge \text{eat}(b, x) \wedge \#x = d]) < 10$

# The semantics of modified numerals

Geurts & Nouwen 2007

According to a simple GQT analysis

[[at least three]] = [[more than two]]. This turns out wrong for several reasons.

## Specificity

- (i) I will invite at most two people, namely Cody and Vic.
- (ii) I will invite fewer than two people, namely Cody and Vic.

# The semantics of modified numerals

Geurts & Nouwen 2007

According to a simple GQT analysis

[[at least three]] = [[more than two]]. This turns out wrong for several reasons.

## Inference

There are 10 marbles in the bag

⇒ There are more than 4 marbles in the bag

?? ⇒ There are at least 4 marbles in the bag

# Tomorrow

Why are there so many different modified numerals?

more than 100 pages

fewer than 100 pages

as many as 100 pages

as few as 100 pages

at least 100 pages

at most 100 pages

no fewer than 100 pages

no more than 100 pages

over 100 pages

under 100 pages

up to 100 pages

100 pages or more

100 pages or fewer

maximally 100 pages

minimally 100 pages

100 pages tops

## References

- Barwise, J. and R. Cooper (1981). Generalized quantifiers and natural language. *Linguistics and Philosophy* 4(2), 159–219.
- van Benthem, J. (1986). *Essays in Logical Semantics*. Dordrecht: Reidel.
- Beyth-Marom, R. (1982). How probable is probable? a numerical translation of verbal probability expressions. *Journal of Forecasting* 1(257–269).
- Bryant, G. and G. Norman (1980). Expressions of probability: words and numbers. *The New England Journal of Medicine* 302(7), 411.
- Erev, I. and B. Cohen (1990). Verbal versus numerical probabilities: Efficiency, biases and the preference paradox. *Organizational Behavior and Human Decision Processes* 45, 1–18.
- Geurts, B. and R. Nouwen (2007). At least et al.: the semantics of scalar modifiers. *Language* 83(3), 533–559.
- Hackl, M. (2000). *Comparative Quantifiers*. Ph. D. thesis, Department of Linguistics and Philosophy, Massachusetts Institute of Technology.
- Heim, I. (2000). Degree operators and scope. In *Proceedings of SALT 10*, Ithaca, NY. CLC Publications.
- Hodges, W. (2001). *Logic: an introduction to elementary logic* (Second Edition ed.). Penguin Books Ltd.
- Horn, L. (2002). Assertoric inertia and NPI licensing. In *CLS 38, Part 2*, pp. 55–82.
- Keenan, E. and J. Stavi (1986). A semantic characterization of natural language determiners. *Linguistics and Philosophy* 9, 253–326.
- Keenan, E. L. (1987). A semantic definition of “indefinite NP”. In E. Reuland and A. ter Meulen (Eds.), *The representation of (in)definiteness*, Chapter 12, pp. 286–317. Cambridge, MA: MIT Press.
- Kennedy, C. (2007). Vagueness and grammar: The semantics of relative and absolute gradable predicates. *Linguistics and Philosophy* 30(1), 1–45.
- Lappin, S. (2000). An intensional parametric semantics for vague quantifiers. *Linguistics and Philosophy* 23, 599–620.
- Milsark, G. (1974). *Existential Sentences in English*. Ph. D. thesis, MIT, Cambridge, MA.
- Montague, R. (1973). The proper treatment of quantification in ordinary english. In K. Hintikka, J. Moravcsik, and P. Suppes (Eds.), *Approaches to Natural Language*, pp. 221–242. Dordrecht: Reidel.
- Montague, R. (1974). *Formal Philosophy: Selected papers of Richard Montague*, Edited by R.H. Thomason. Yale University Press.
- Moxey, L. (1986). *A psychological investigation of the use and interpretation of English quantifiers*. Ph. D. thesis, University of Glasgow.

- Moxey, L. (2006). Effects of what is expected on the focussing properties of quantifiers: a test of the presupposition-denial account. *Journal of Memory and Language* 55, 422–439.
- Moxey, L. and A. Sanford (2000). Communicating quantities: a review of psycholinguistic evidence of how expressions determine perspectives. *Applied Cognitive Psychology* 14, 237–255.
- Newstead, S. and K. Coventry (2000). The role of context and functionality in the interpretation of quantifiers. *European Journal of Cognitive Psychology* 12(2), 243–259.
- Nouwen, R. (2003a). Complement anaphora and interpretation. *Journal of Semantics* 20(1), 73–113.
- Nouwen, R. (2003b). *Plural pronominal anaphora in context*. Number 84 in Netherlands Graduate School of Linguistics Dissertations. Utrecht: LOT.
- Nouwen, R. (2006). Remarks on the polar orientation of ‘almost’. In J. van de Weijer and B. Los (Eds.), *Linguistics in the Netherlands*. Benjamins.
- Renooij, S. and C. Witteman (1999). Talking probabilities: communicating probabilistic information with words and numbers. *International Journal of Approximate Reasoning* 22, 169–194.
- Sanford, A., N. Fay, A. Stewart, and L. Moxey (2002). Perspective in statements of quantity, with implications for consumer psychology. *Psychological Science* 13, 130–134.
- Sanford, A. and L. Moxey (1993). *Communicating quantities. A psychological perspective*. Laurence Erlbaum Associates.
- Wallsten, T., D. Budescu, A. Rapoport, R. Zwick, and B. Forsyth (1986). Measuring the vague meanings of probability terms. *Journal of Experimental Psychology: General*.
- Wright, D., G. Gaskell, and C. O’Muircheartaigh (1995). Testing the multiplicative hypothesis of intensifiers. *Applied Cognitive Psychology* 9, 167–177.