

# Scales, quantity & degree

## **Lecture 2: Numerals and modification**

Rick Nouwen (Utrecht)

October 2013

more than 100 pages

fewer than 100 pages

at least 100 pages

at most 100 pages

more than 100 pages

#pp > 100

fewer than 100 pages

#pp < 100

at least 100 pages

#pp ≥ 100

at most 100 pages

#pp ≤ 100

more than 100 pages

fewer than 100 pages

more than 100 pages

fewer than 100 pages

as many as 100 pages

as few as 100 pages

more than 100 pages  
fewer than 100 pages

as many as 100 pages  
as few as 100 pages

at least 100 pages  
at most 100 pages

more than 100 pages  
fewer than 100 pages

at least 100 pages  
at most 100 pages

as many as 100 pages  
as few as 100 pages

no fewer than 100 pages  
no more than 100 pages

























Sir Francis Galton, 1881,

Visualised Numbers, *Journal of the Anthropological Institute* **10**

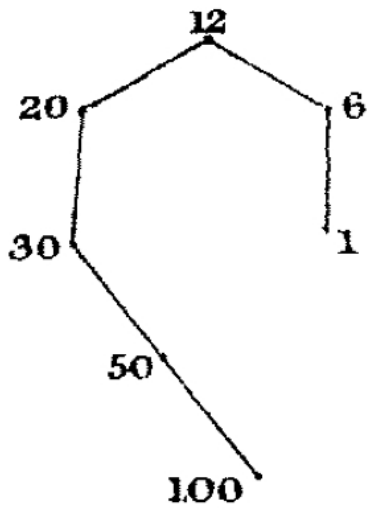


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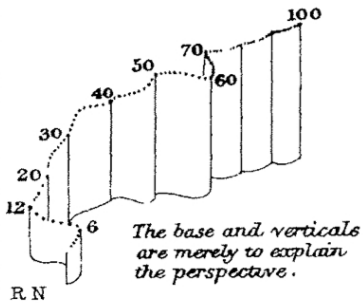
Visualised Numbers, *Journal of the Anthropological Institute* **10**

*The representation I carry in my mind of the numerical series is quite distinct to me, so much so that I cannot think of any number but I at once see it (as it were) in its peculiar place in the diagram.*

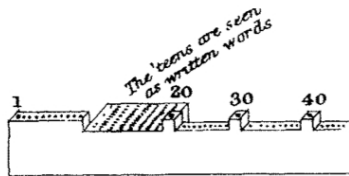
T.M.



by G H

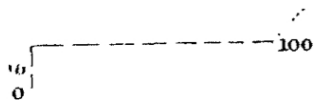


R N

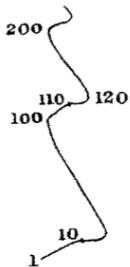


The frame is merely to explain the perspective.

E.D



T H



M E.H

# Number forms / lines

- *Number forms*: “this peculiarity consist in the sudden and automatic appearance of a vivid and invariable *Form* in the mental field of view, whenever a numeral is thought of, and in which each numeral has its own definite place.”

(Galton 1881, p.88)

- type of synesthesia
- occurrence:  $\pm 12\%$  (Ward et al. 2009, *Cortex* **45**)
- people who have a number form are or at least can become aware of the fact that they have it

# The SNARC

- Spatial-Numerical Association of Response Codes

(Stanislas Deheane et al. 1993)

- In contrast to *number forms*, the SNARC is subconscious
- Task: decide whether a given number (0-9) is odd or even
- Response rule 1: left button → odd, right button → even
- Response rule 2: left button → even, right button → odd
- 3 quicker responses with rule 1
- 7 quicker responses with rule 2

# The metaphor of the heap

- Lakoff & Johnson 1980: MORE IS UP; LESS IS DOWN
- the metaphor of quantity as a pile of stuff
- not a psychological, but a linguistic relation between space and number
- not horizontal, but vertical

my income rose

the number of errors he made is low

he is underage

his blood pressure is high

# The metaphor of the heap in modified numerals

over 100 pages

under 100 pages

up to 100 pages



# The metaphor of the heap in modified numerals

over 100 pages

under 100 pages

up to 100 pages

\*next to 100 pages

\*in front of 100 pages

\*behind 100 pages

# The metaphor of the heap in modified numerals

over 100 pages

American English:

under 100 pages

He is earning north of \$50.000

up to 100 pages

\*next to 100 pages

\*in front of 100 pages

\*behind 100 pages

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Possible exception:

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John earns around \$50.000

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Possible exception:

\*next to 100 pages

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\*in front of 100 pages

shàngxià

up-down (vertical)

zuǒyòu

left-right (horizontal)

\*behind 100 pages

and qiánhòu

front-behind (lateral)

# The metaphor of the heap in modified numerals

- Corver & Zwarts 2006: cross-linguistically, prepositions that modify numerals always have a sense that is compatible with vertical orientation

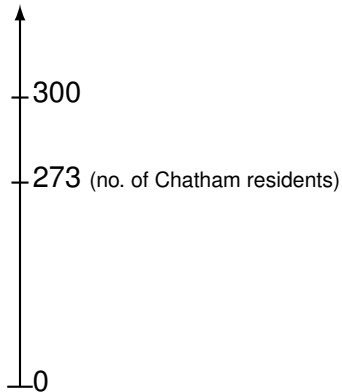
between 100 and 200 pages

north of \$100,000

- Parts of the spatial semantics of prepositions is apparently to relevant their non-spatial semantics



The cat is under the table



Chatham has under 300 residents.

## Further spatial semantics in modified numerals

Chatham has 273 inhabitants.

and, so, Chatham has under 300 residents.

Chatham has 273 inhabitants.

#and, so, Chatham has up to 300 residents.



## Further spatial semantics in modified numerals

I logged in to my twitter account yesterday and was disappointed to find out that I still have under 1000 followers.

??I logged in to my twitter account yesterday and was disappointed to find out that I still have up to 1000 followers.

## End of warm-up: Questions

- How superficial are the spatial aspects of prepositional modified numerals?
- Is the locative / directional distinction relevant to modified numerals?
- How general is this correspondence between the modified numeral domain and the domain from which the modifier originates?

- 1 Warm-up: preliminary thoughts on number and space
- 2 Two kinds of modified numerals**
- 3 The class A / class B distinction
- 4 Analysis 1
- 5 Analysis 2
- 6 Beyond the A/B distinction

# Ignorance

Generalisation (to be refined):

*More than  $n$* : compatible with speaker ignorance

*At least  $n$* : requires speaker ignorance

# Ignorance

## Generalisation (to be refined):

*More than n*: compatible with speaker ignorance

*At least n*: requires speaker ignorance

(1) I have more than 1 child.

(2) I have at least 2 children.

# Ignorance

## Generalisation (to be refined):

*More than n*: compatible with speaker ignorance

*At least n*: requires speaker ignorance

- (1) I have more than 1 child.
- (2) I have at least 2 children.
- (3) More than 50 people turned up for the party. 62, to be precise.
- (4) At least 50 people turned up for the party. #62, to be precise.

# Ignorance

## Generalisation (to be refined):

*More than n*: compatible with speaker ignorance

*At least n*: requires speaker ignorance

- (1) I have more than 1 child.
- (2) I have at least 2 children.
- (3) More than 50 people turned up for the party. 62, to be precise.
- (4) At least 50 people turned up for the party. #62, to be precise.
- (5) I know exactly how much memory my laptop has ...
  - a. ... and it's more than 2GB.
  - b. ... #and it's at least 2GB.

## Generalisation

*At least/most  $n$*  requires speaker ignorance.

However, this requirement can be obviated in particular environments.



- (6) (Sales person:) All our laptops have at least 2GB of memory.

Reading 1: I forgot how much memory our laptops have, but I'm sure it's not less than 2GB.

Reading 2: the slowest laptop we sell has 2GB of memory, but we sell faster ones too

## Generalisation

*At least/most  $n$*  requires speaker ignorance.

However, this requirement can be obviated in particular environments.

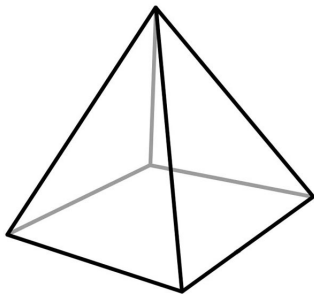
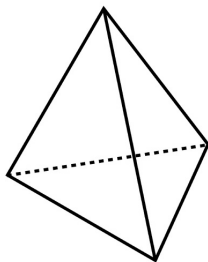
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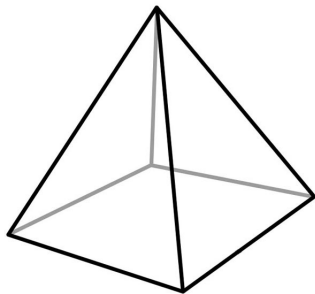
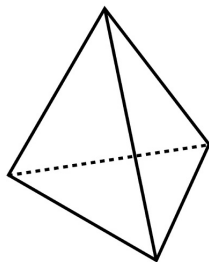
- (8) A pyramid has at least three outer triangular surfaces.

- (8) A pyramid has at least three outer triangular surfaces.



# Data

- (8) A pyramid has at least three outer triangular surfaces.



- (9) #A triangle has at most 10 sides.  
(10) A triangle has fewer than 10 sides.

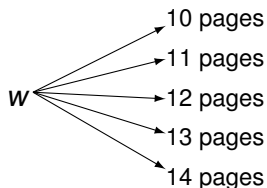
# Data

- (11) John's paper has to be at least 20 pages long.
  - a. Ignorance: I'm not sure what the lower page limit for John's paper is, but it is not under 20.
  - b. Authority: The lower page limit for John's paper is 20.
  
- (12) John's paper is allowed to be at most 20 pages long.
  - a. Ignorance: I'm not sure what the upper page limit for John's paper is, but it is not over 20.
  - b. Authority: The upper page limit for John's paper is 20.
  
- (13) John's paper is at least/most 20 pages long.
  - Ignorance only

# Data

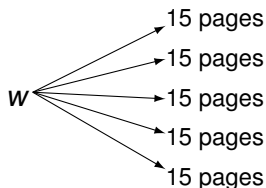
(14) John's paper has to be at least 10 pages long.

(15) John's paper has to be more than 9 pages long.



(14): true

(15): true



(14): false

(15): true



# Ignorance and Ignorance Obviation

- (16) John ate an apple or a pear.

Ignorance only

- (17) John must eat an apple or a pear.

Ignorance: I forgot what fruit John must eat, but it is either an apple or it is a pear.

Authority: John can meet the requirement by eating an apple and he can meet the requirement by eating a pear.

- (18) *Irgendeine Student hat angerufen.*

*irgendeine* student has called.

'Some student called (and I don't know/care who).'

- (19) *Maria muss irgendeinen Arzt heiraten.*

Maria must *irgendeinen* doctor marry.

'Maria should marry a doctor, any doctor is a permissible option'

**Q: What mechanism is responsible for ignorance/obviation?**

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- 1** The form of the quantifier determines the effects

Nouwen 2010, Penka 2010

- 2** The relation expressed by the quantifier determines the effects

Inspired by Buring 2008, Nilsen 2007, Cummins & Katsos 2010, Schwarz 2013

- 3** Combining form and Function

Nouwen 2013

First: regularities in the data

- 1 Warm-up: preliminary thoughts on number and space
- 2 Two kinds of modified numerals
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- 4 Analysis 1
- 5 Analysis 2
- 6 Beyond the A/B distinction

Languages tend to distinguish two kinds of modified numerals

**Class A:** no ignorance requirement, no ignorance obviation

**Class B:** ignorance requirement, obviated in particular environments

# Data

Nouwen 2010: the class A/B distinction

- (20) I have up to 10 children.
- (21) Up to 3,000 people attended a special ceremony last night to witness torchbearers create the Paralympic Flame.
- (22) (Sales person:) Our laptops have up to 5GB of memory.
- (23) He could face up to 3 years in prison.

# Data

Nouwen 2010: the class A/B distinction

## English

<b>A</b>	<b>B</b>
more than $n$ less/fewer than $n$ over $n$ under $n$ between $n$ and $m$	at least $n$ at most $n$ up to $n$ from $n$ from $n$ to $m$ minimally $n$ maximally $n$ $n$ or more $n$ or fewer

# Data

Nouwen 2010: the Class A/B distinction

## Dutch

<b>A</b>	<b>B</b>
meer dan $n$ minder dan $n$ boven de $n$ onder de $n$ tussen de $n$ en de $m$	tenminste $n$ hoogstens $n$ tot $n$ vanaf $n$ van $n$ tot $m$ minimaal $n$ maximaal $n$ $n$ of meer $n$ of minder



# Data

## Nouwen 2010: the Class A/B distinction

- (24) *Un triangolo ha piu di 1 lato*  
A triangle has more than 1 side
- (25) *#Un triangolo ha almeno 2 lati.*  
A triangle has at least 2 sides
- (26) a. *10-nin yori ooku-no seito-ga jugyoo-ni*  
10-CL from many-Gen students-Nom class-to  
*kita.*  
came.
- b. *Jissai, 12-nin-no seito-ga kita.*  
in fact 12-CL-Gen students-Nom came.  
“More than 10 students came to the class. In fact,  
12 did.”
- (27) a. *saitei 10-nin-no seito-ga jugyoo-ni kita.*  
least 10-CL-Gen students-Nom class-to came.
- b. *#Jissai, 12-nin-no seito-ga kita.*  
in fact 12-CL-Gen students-Nom came.  
“At least 10 students came to the class. #In fact,  
12 did.”

## Class B expressions

at least  
at most  
up to  
from ... to  
maximally  
minimally

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# Analysis 1

Nouwen 2010

**Class B modified numerals are end-point markers.  
They presuppose they operate on a range, i.e. a  
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(28) #The tallest queen of the Netherlands is called Beatrix.

(30) #The maximum number of wheels on my car is 4.

(32) #John is standing up to this line.

**Class B modified numerals are end-point markers.  
They presuppose they operate on a range, i.e. a  
non-singleton set of values.**

- (28) #The tallest queen of the Netherlands is called Beatrix.
- (29) The tallest queen the Netherlands ever had is called Beatrix.
- (30) #The maximum number of wheels on my car is 4.
- (31) The maximum no. of wheels I can fit on the back seat is 4.
- (32) #John is standing up to this line.
- (33) John drove up to this line (and then stopped)

# Minimum / Maximum

Nouwen 2010

- (i) #A triangle has minimally 2 sides
- (ii) #A triangle has maximally 10 sides

Reasonable assumption: the operators *min* and *max* presuppose a range.

- $\min(\{3\})$
- $\max(\{3\})$
- $\min(\{3, 5, 8\})$

# Avoidance of presupposition failure with superlatives

- (i) The highest amount of prize money that Federer won so far is \$1 million.  
That was for Wimbledon 2009.  
the amount of prize money that Federer won in some tournament
  
- (ii) The highest amount of prize money that Federer won so far is \$8.3 million.  
That was over 2006.  
the amount of prize money that Federer won in some year



# Directionality and paths

Jasper is standing in front of the goal line  
??Jasper is standing up to the goal line

Jasper arrive before 10PM  
??Jasper arrived until 10PM

Chatham has under 300 residents  
Chatham has up to 300 residents

# Directionality across domains

- (??)Your paper is up to 12 pages long
- ??Jasper is standing up to the goal line
- ??Jasper arrived until 10PM

# Directionality across domains

(??)Your paper is up to 12 pages long  
??Jasper is standing up to the goal line  
??Jasper arrived until 10PM

Your paper is allowed to be up to 12 pages long  
Jasper is allowed to stand up to the goal line  
Jasper is allowed to arrive until 10PM

# Directionality across domains

(??)Your paper is up to 12 pages long  
??Jasper is standing up to the goal line  
??Jasper arrived until 10PM

Your paper is allowed to be up to 12 pages long  
Jasper is allowed to stand up to the goal line  
Jasper is allowed to arrive until 10PM

tot 12 pagina's  
tot de doellijn  
tot 10 uur 's avonds

# Homogeneity

- Path-expressions share a homogeneity requirement

#Jasper ate an apple until 3PM.

Jasper slept until 3PM.

- Bennett & Partee 1972: 'Jasper slept' holds at interval  $I$  and 'Jasper slept' holds at each sub-interval  $I'$  of  $I$ .

# Spatial homogeneity

#He relocated up to Amsterdam.

#The sign points up to the auditorium.

#I crossed up to the North side of the pass.

(Pinon 1994)

Explanation: these are relations between two points, no homogeneous scales.

# Numeral homogeneity

Jasper is allowed to invite fewer than 10 friends.

⇒ Jasper is allowed to invite 9 friends.

Jasper is allowed to invite up to 9 friends.

⇒ Jasper is allowed to invite 9 friends.

[[at most 20]]

=

[[up to 20]]

=

[[maximally 20]]

=

$$\lambda D_{\langle d,t \rangle} : |D| > 1.\max(D) = 20$$



# Analysis 1

Nouwen 2010

John's paper is allowed to be at most 20 pages long.

$$A \mid \max(\lambda d. \diamond[\text{John's paper is } d \text{ pages long}]) = 20$$

$$P \mid |\lambda d. \diamond[\text{John's paper is } d \text{ pages long}]| > 1$$

(authority reading + weak deontic free choice)

(I use  $\diamond$  for deontic versus  $\lozenge$  for epistemic modality)

# Analysis 1

Nouwen 2010

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Presupposition failure: this is a singleton set

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Optional: *at most 20* takes scope over covert epistemic operator

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(ignorance reading + weak epistemic free choice)

# Analysis 1

Nouwen 2010

(I'm not sure what the page limit for the paper is, but)  
John's paper is allowed to be at most 20 pages long.

$$A \mid \max(\lambda d. \diamond \diamond [\text{John's paper is } d \text{ pages long}]) = 10$$

$$P \mid |\lambda d. \diamond \diamond \text{John's paper is } d \text{ pages long}| > 1$$

(ignorance reading + weak epistemic free choice)

# Analysis 1

Nouwen 2010

- Class B modifiers presuppose a non-singleton set argument
- Advantage 1: straightforward account of what the A/B distinction looks like
- Advantage 2: accounts for weak free choice effects
- Downside: need to posit optional presence of covert modals
- Bigger problem: *at least*

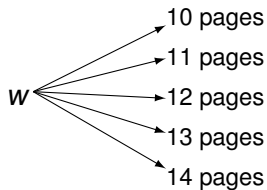
# Analysis 1

Nouwen 2010

John's paper needs to be at least 10 pages long.

A |  $\min(\lambda d. \Box \text{John's paper is } d \text{ pages long}) = 10$

P |  $|\lambda d. \Box \text{John's paper is } d \text{ pages long}| > 1$



$\lambda d. \Box [\text{John's paper is } d \text{ pages long}] = \emptyset$

## Possible ways out:

- Blame the problem on the modal
  - Modified numerals restrict modal base (Nouwen 2010)
  - Modals can embed speech acts; modified numerals can take scope inbetween modal and speech act (Cohen & Krifka 2012)
- ! Every paper received at least three reviews from PC members
- Blame it on degree semantics (Spector 2011, Fox 2011)
  - More promising, but not without problems either

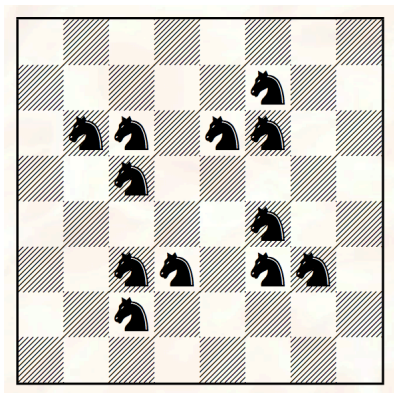


## Interlude: minimum requirements

The smallest number of knights needed to occupy or attack every square on the chess board is 12

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## Interlude: minimum requirements

One-half of a road construction project was completed by 6 workers in 12 days. Working at the same rate, what is the smallest number of workers needed to finish the rest of the project in exactly four days?

**Answer:**

The smallest number of workers needed to finish the project in 4 days is 18.

## Interlude: minimum requirements

- Expressions of minimum requirement are frequent
- Expressions of minimum requirement are easy to interpret
- Yet, standard theories do not properly account for their meaning
- The problem is exactly the problem we encountered with **at least**

# Minimum requirements: the puzzle

One-half of a road construction project was completed by 6 workers in 12 days.

Working at the same rate, what is the smallest number of workers needed to finish the rest of the project in exactly four days?

- *To finish in 4 days, you need 18 workers* (true)
- every world in which the project finishes in 4 days is a world in which 18 workers are on the project
- *To finish in 4 days, you need workers* (true)
- every world in which the project finishes in 4 days is a world in which there are workers on the project

# Minimum requirements: the puzzle

One-half of a road construction project was completed by 6 workers in 12 days.

Working at the same rate, what is the smallest number of workers needed to finish the rest of the project in exactly four days?

- *To finish in 4 days, you need 18 workers* (true)
- *To finish in 4 days, you need workers* (true)
- So what is the minimum number of workers, such that every world in which the project finishes in 4 days has so many workers on the project?

# Minimum requirements: the puzzle

One-half of a road construction project was completed by 6 workers in 12 days.

Working at the same rate, what is the smallest number of workers needed to finish the rest of the project in exactly four days?

- *To finish in 4 days, you need 18 workers* (true)
- *To finish in 4 days, you need workers* (true)
- So what is the minimum number of workers, such that every world in which the project finishes in 4 days has so many workers on the project?
- *The smallest number of workers needed to finish the project in 4 days is 1* (oops)

# Minimum requirements: the puzzle

One-half of a road construction project was completed by 6 workers in 12 days.

Working at the same rate, what is the smallest number of workers needed to finish the rest of the project in exactly four days?

- *To finish in 4 days, you need exactly  $n$  workers (false for any  $n$ )*



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## Analysis 2

Büring 2008, Nilsen 2007, Cummins & Katsos 2010

Class B modified numerals are disjunctive

at least 10 X Y  $\sim$  [exactly 10 X Y]  $\vee$  [more than 10 X Y]

Analysis of class B effects are fully parallel to analysis of disjunction

(34) John ate an apple or a pear.

(35) John may eat an apple or a pear.

# Disjunction, ignorance, obviation

John ate an apple or a pear.

ignorance

John may eat an apple or a pear.

obviation

(Free choice, Kamp 1974)

John is required to eat an apple or a pear.

obviation

John invited at most 20 friends.

ignorance

John may invite at most 20 friends.

obviation

John is required to invite at most 20 friends.

obviation

# The standard approach to implicature

Scale:  $p \models q$

A:  $q$  *(assertion)*

PI:  $\neg Bp$  *(primary implicature)*

C:  $(Bp \vee B\neg p) \wedge (Bq \vee B\neg q)$  *(competence)*

SI:  $B\neg p$  *(secondary implicature = PI + C)*

# Disjunction and ignorance implicature

e.g. Sauerland 2004, Geurts 2012

Scale:  $p \models p \vee q$   
 $q \models p \vee q$   
 $(p \wedge q \models p \vee q)$

A:  $p \vee q$

PI:  $\neg Bp, \neg Bq$

C:  $(Bp \vee B\neg p) \wedge (Bq \vee B\neg q)$

SI:  $B\neg p \wedge B\neg q \quad \rightsquigarrow$  contradicts A

It follows that  $\neg C$ : the speaker is ignorant about which of the disjuncts is true

# Obviation

Scale:  $\Box p \models \Box(p \vee q)$   
 $\Box q \models \Box(p \vee q)$   
 $(\Box(p \wedge q) \models \Box(p \vee q))$

A:  $\Box(p \vee q)$

PI:  $\neg B\Box p, \neg B\Box q$

C:  $(B\Box p \vee B\neg\Box p) \wedge (B\Box q \vee B\neg\Box q)$

SI:  $B\neg\Box p \wedge B\neg\Box q \quad \rightsquigarrow$  does not contradict A

It follows that:  $B\Diamond p \wedge B\Diamond q \wedge B\Box(p \vee q)$

# Modified numerals and ignorance implicature

Schwarz 2013

Scale:  $x = 3 \models x \geq 3$   
 $x > 3 \models x \geq 3$

A:  $x \geq 3$

PI:  $\neg B(x = 3), \neg B(x > 3)$

C:  $(B(x = 3) \vee B(x \neq 3)) \wedge (B(x > 3) \vee B(x \geq 3))$

SI:  $B(x \neq 3) \wedge B(x \geq 3) = B(x < 3) \quad \leadsto$  contradicts A

It follows that the speaker believes that  $x \geq 3$  but that she does not know whether  $x = 3$  or  $x > 3$ .

# Modified numerals and obviation

Schwarz 2013

Scale:  $\Box(x = 3) \models \Box(x \geq 3)$   
 $\Box(x > 3) \models \Box(x \geq 3)$

A:  $\Box(x \geq 3)$

PI:  $\neg B\Box(x = 3), \neg B\Box(x > 3)$

C:  $(B\Box(x = 3) \vee B\Box(x \neq 3)) \wedge (B\Box(x > 3) \vee B\Box(x \geq 3))$

SI:  $B\Box(x \neq 3) \wedge B\Box(x \geq 3) = B\Box(x < 3) \quad \rightsquigarrow$  does not contradict A

It follows that the speaker believes that  $\Box(x \geq 3)$  and that  $\Diamond(x = 3)$  and  $\Diamond(x > 3)$



## class B quantifiers as disjunctions: summary

- Class B expressions express  $\leq / \geq$
- Such non-strict comparison relations are disjunctive
- Disjunction give rise to ignorance implicatures and obviation patterns
- Now for some complications

# complication 1: alternatives

The reasoning is too general

Scale:  $x = 3 \models x \geq 3$   
 $x > 3 \models x \geq 3$

Scale:  $x = 4 \models x > 3$   
 $x > 4 \models x > 3$

# Possible counter objection

## Geurts et al. 2010:

Berta had 3 beers $\Rightarrow$ Berta had at least 3 beers	50%
Berta had 3 beers $\Rightarrow$ Berta had more than 2 beers	100%

## Cummins & Katsos: Symbolic version of Geurts et al.

A A A A A A A

A  $\geq$  6

A > 6

etc.

$\geq$  significantly longer RT  
than >

## complication 2: $\geq$ without ignorance

- not all expressions of  $\leq / \geq$  give rise to ignorance implicatures
- equatives (**as many as**)
- Chinese and Japanese prepositional numerals
- negative differentials (**no more than**)  
 $\neg(x > 30) = x \leq 30$

## complication 2: $\geq$ without ignorance

*#Sanjiaoxing zui-shao you liang-tiao bian.*  
triangle      most-little have 2-CL      side

$3 \geq 2$

(native speaker judgement: this indicates a minimum)

*Sanjiaoxing zhi-shao you liang-tiao bian.*  
triangle      to-little have 2-CL      side

$3 \geq 2$

## complication 2: $\geq$ without ignorance

John invited no fewer than 55 friends.

Reading: *John invited exactly 55 (and that's a lot)*

- This looks like a strong quantity implicature
- $\langle \dots, x \leq 54, x \leq 55, x \leq 56, \dots \rangle$
- Once again: we need to say something about where the alternatives come from  
compare to Fox & Hackl 2007, Nouwen 2008

## complication 3: $\leq$

Your paper may be at most 10 pages long

A:  $\diamond(x \leq 10)$

PI:  $\neg B \diamond(x = 10), \neg B \diamond(x < 10)$

SI:  $B \neg \diamond(x = 10) \wedge B \neg \diamond(x < 10)$

contradicts A

Target meaning:  $B \neg \diamond(x > 10)$

## Analysis 2

- The disjunction approach can use well-known mechanisms of ignorance implicature and obviation
- But the a stipulation is needed which forms are associated with the same scale as that of disjunction
- No mechanism for upper bounds
  
- Perhaps the comparison with disjunction is misleading
- maybe we should look elsewhere



# A slightly different take on analysis 2

Nouwen 2013

- (36) *Irgendeine Student hat angerufen.*  
*irgendeine* student has called.  
'Some student called (and I don't know/care who).'
- (37) *Maria muss irgendeinen Arzt heiraten.*  
Maria must *irgendeinen* doctor marry.  
'Maria should marry a doctor, any doctor is a permissible option'

Kratzer & Shimoyama 2002, Aloni & Port 2012

# A slightly different take on analysis 2

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Kratzer & Shimoyama 2002, Aloni & Port 2012

- (38) *Juan está en alguna habitación de la case.*  
Juan is in *alguna* room of the house.  
'Juan is in a room in the house (and I don't know which)'
- (39) *Juan tiene estar en alguna habitación de la case.*  
Juan has to be in *alguna* room of the house.  
'Juan has to be in a room in the house'

## Alonso-Ovalle & Benito-Menéndez 2010

- The epistemic indefinite *algún* presupposes that the domain of quantification is not a singleton
- This presupposition triggers epistemic implicatures

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## Analysis 1: range operators

- Class B modified numerals presuppose they operate on a range, i.e. a non-singleton set.
- Optional covert epistemic modality triggers ignorance reading

# Reasoning about anti-specificity

Kratzer & Shimoyama 2002, Alonso-Ovalle & Benito-Menendez

Irgendeine student hat angerufen

A:  $\exists x[x \in f(\textit{student}) \wedge \textit{called}(x)]$

P:  $|f(\textit{student})| > 1$

I:  $\neg \exists y[\textit{student}(y) \wedge B \exists x[x \in \{y\} \wedge \textit{called}(x)]]$

- The speaker has used an anti-specific expression
- Why did s/he not use something with a narrower domain?
- It must be because there is no narrower for which the speaker believes what she said to be true

# Anti-specificity in modified numerals

Nouwen 2013

## John read at least 3 books

A:  $|\lambda x.read(j, x) \wedge book(x)| \in f(\{n|n \geq 3\})$

P:  $|f(\{n|n \geq 3\})| > 1$

I:  $\neg \exists n \geq 3 B[|\lambda x.read(j, x) \wedge book(x)| \in \{n\}]$

Various problems:

- I is too strong
- Still no upper bounds
- Obviation is slightly more difficult

# Taking stock

## Analysis 1

- Class B expressions impose an anti-singleton constraint
- Ignorance by covert modal
- Problem for *at least*

## Analysis 2

- Class B expressions impose an anti-singleton constraint
- Ignorance by implicature
- Problem for *at most*

# The A/B distinction

## English

<b>A</b>	<b>B</b>
more than $n$ less/fewer than $n$ over $n$ under $n$ between $n$ and $m$	at least $n$ at most $n$ up to $n$ from $n$ from $n$ to $m$ minimally $n$ maximally $n$ $n$ or more $n$ or fewer

Criterion: compatibility with specific amounts



# Finer distinctions within class B

Schwarz et al. 2012

## NPI licensing

At most 10 people have ever seen this

\*Up to 10 people have ever seen this

## The bottom of the scale effect

At most one person died in the crash

??Up to one person died in the crash

Use up to one kg of sugar to make this jam

# Finer distinction within class B

Blok 2013

## Upper bound strength

At most 100 people have read my paper. #Maybe even more

Up to 100 people have read my paper. Maybe even more

## Perspective

Fortunately, at most 10 people got injured

#Fortunately, up to 10 people got injured

# Finer distinctions within class B

Blok 2013

- Directionals do not license NPIs
- Directionals do not combine with the bottom of the scale
- Directionals have a weak upper bound
- Directionals have a positive perspective

This pattern is cross-linguistically stable for directionals: Dutch, English, German, Italian, Chinese, Turkish, Danish, Greek, Farsi, Romanian, Polish, Hungarian, Spanish and Russian.

# Future direction: what is directionality anyway

Blok 2013

\*Er hoeven tot 200 flessen wijn besteld te worden.

\*Er hoeven tegen de 200 flessen wijn besteld te worden.

Ik heb tegen de honderd uitnodigingen verstuurd      proximity

Hij liep tegen de muur      end-point

Hij leunde tegen de muur      contact

## Further future directions (ERC project)

- Cross-linguistic stability of generalisations
- The nature of ignorance and obviation effects