# A Proof-Theoretic Universal Property of Determiners

*Determiners* are the natural language (NL) analogue of quantifiers in logic. In model-theoretic semantics (MTS), their denotations are taken as binary relation over subsets of the domain of the model (see [3] for an extensive discussion). When combined with a noun-meaning, a subset of the domain, they give rise to *determiner phrase* (*dp*), which, according to the *generalised quantifiers* theory [1], a cornerstone of MTS, denotes a *generalised quantifier* (GQ), a subset of the power set of the domain $\mathcal{D}$. It is generally *assumed* (and empirically attested), that the only GQs that can be denotation of *dp*s are the *conservative* ones, satisfying, for every $X, Y \subseteq \mathcal{D}$: $[\![D]\!](X)(Y)$ iff $[\![D]\!](X)(X \cap Y)$.
Example: every girl smiles iff every girl is a girl who smiles.
Since there exist many non-conservative binary relation on subsets of a domain, conservativity serves as a *selection criterion* for possible determiner denotations.

On the other hand, in *proof-theoretic semantics* (PTS), an approach to semantics according to which meaning is determined by means of the rules of a meaning-conferring natural-deduction (ND) proof-system (see [4] as a general reference and [references suppressed] for PTS for NL), independently of models and truth-conditions. Thus meaning is captured by *inferential role.* In [reference suppressed] it is suggested that the proof-theoretic meaning of a determiner $D$ is the following (with some details suppressed): $[\![D]\!] = \lambda z_1 \lambda z_2 \lambda \Gamma . \bigcup_{\mathbf{j}_1, \ldots, \mathbf{j}_m \in \mathcal{P}} I_D(z_1)(z_2)(\mathbf{j}_1) \cdots (\mathbf{j}_m)(\Gamma)$, where the notation means (with more detail in the paper):
– $z_1$ ranges over (proof-theoretic) noun meanings and $z_2$ over (proof-theoretic) *vp* meanings.
– $\Gamma$ is a collection of NL sentences, from which a conclusion sentence $S$ including a *dp* headed by $D$ (here, in subject position only, for simplicity) can be inferred.
–The $\mathbf{j}_k$s are *individual parameters.*
– $I_D$ is the introduction-rule ($I$-rule) for $D$ in the meaning-conferring ND-system. The *dp* headed by $D$ is introduced into the subject position of $S$ similarly to introducing a connective or quantifier into a logical formula, and similarly for elimination.
Based on this characterisation of determiners' meanings and on a suitable adaptation of conservativity to a proof-theoretic setting, it was proved [reference suppressed] that *every determiner is conservative.*

Thus, conservativity cannot serve as a PTS criterion for classifying determiners meanings. In this paper, I want to argue for another classifying property of determiners meanings, based on their proof-theoretic characterization. This criterion is the well-known *harmony* property of the $I/E$-rules, the absence of which disqualifies an ND-system from being considered as meaning-conferring.
The paper is structured as follows.
– A presentation of a simplification of the extensional fragment of English and its accompanying ND meaning-conferring proof-system, in terms of which the issue is discussed.
– A definition of *harmony* [2], and proof of harmony of the meaning-conferring rules for the fragment's determiners every, some and no.
– A proof-theoretic definition of a "pathological" determiner, *donk*, by means of $I/E$-rules, that in spite of being conservative cannot be an admissible NL determiner.
– A proof of the *disharmony* of the $I/E$-rules for *donk*, that in terms of truth-conditions, the effect of *donk* in a sentence like *donk* girl smiles, is that either every girl smiles or no girl smiles, a trivialising effect.
– Conclusion: harmony of a determiner's $I/E$-rules is a necessary condition for the determiner being NL-admissible.

## References


[1] Jon Barwise and Robin Cooper. Generalized quantifiers and natural language. *Linguistics and Philosophy*, 4(2):159–219, 1981.

[2] Michael Dummett. *The Logical Basis of Metaphysics.* Harvard University Press, Cambridge, MA., 1993 (paperback). Hard copy 1991.

[3] Stanley Peters and Dag Westerståhl. *Quantifiers in Language and Logic*. Oxford University Press, 2006.

[4] Peter Schroeder-Heister. Proof-theoretic semantics. In Eduard N. Zalta, editor, *Stanford Encyclopaedia of Philosophy (SEP), http://plato.stanford.edu/*. The Metaphysics Research Lab, Center for the Study of Language and Information, Stanford University, Stanford, CA, 2011.