A Proof-Theoretic Universal Property of Determiners

Determiners are the natural language (NL) analogue of quantifiers in logic. In model-theoretic semantics (MTS), their denotations are taken as binary relation over subsets of the domain of the model (see [3] for an extensive discussion). When combined with a noun-meaning, a subset of the domain, they give rise to determiner phrase (dp), which, according to the generalised quantifiers theory [1], a cornerstone of MTS, denotes a generalised quantifier (GQ), a subset of the power set of the domain \( D \). It is generally assumed (and empirically attested), that the only GQs that can be denotation of dps are the conservative ones, satisfying, for every \( X, Y \subseteq D \): \([D](X) (Y) \) iff \([D](X) (X \cap Y)\).

Example: every girl smiles iff every girl is a girl who smiles.

Since there exist many non-conservative binary relation on subsets of a domain, conservativity serves as a selection criterion for possible determiner denotations.

On the other hand, in proof-theoretic semantics (PTS), an approach to semantics according to which meaning is determined by means of the rules of a meaning-conferring natural-deduction (ND) proof-system (see [4] as a general reference and [references suppressed] for PTS for NL), independently of models and truth-conditions. Thus meaning is captured by inferential role. In [reference suppressed] it is suggested that the proof-theoretic meaning of a determiner \( D \) is the following (with some details suppressed): \([D] = \lambda z_1 \lambda z_2 \lambda \Gamma . \bigcup_{j_1, \ldots, j_m \in P} I_D(z_1)(z_2)(j_1) \cdots (j_m)(\Gamma)\), where the notation means (with more detail in the paper):

- \( z_1 \) ranges over (proof-theoretic) noun meanings and \( z_2 \) over (proof-theoretic) vp meanings.
- \( \Gamma \) is a collection of NL sentences, from which a conclusion sentence \( S \) including a dp headed by \( D \) (here, in subject position only, for simplicity) can be inferred.
- The \( j_k \)'s are individual parameters.
- \( I_D \) is the introduction-rule (I-rule) for \( D \) in the meaning-conferring ND-system. The dp headed by \( D \) is introduced into the subject position of \( S \) similarly to introducing a connective or quantifier into a logical formula, and similarly for elimination.

Based on this characterisation of determiners' meanings and on a suitable adaptation of conservativity to a proof-theoretic setting, it was proved [reference suppressed] that every determiner is conservative. Thus, conservativity cannot serve as a PTS criterion for classifying determiners meanings. In this paper, I want to argue for another classifying property of determiners meanings, based on their proof-theoretic characterization. This criterion is the well-known harmony property of the \( I/E \)-rules, the absence of which disqualifies an ND-system from being considered as meaning-conferring.

The paper is structured as follows.

- A presentation of a simplification of the extensional fragment of English and its accompanying ND meaning-conferring proof-system, in terms of which the issue is discussed.
- A definition of harmony [2], and proof of harmony of the meaning-conferring rules for the fragment’s determiners every, some and no.
- A proof-theoretic definition of a “pathological” determiner, donk, by means of \( I/E \)-rules, that in spite of being conservative cannot be an admissible NL determiner.
- A proof of the disharmony of the \( I/E \)-rules for donk, that in terms of truth-conditions, the effect of donk in a sentence like donk girl smiles, is that either every girl smiles or no girl smiles, a trivialising effect.
- Conclusion: harmony of a determiner’s \( I/E \)-rules is a necessary condition for the determiner being NL-admissible.

References

