The ruler model of granularity

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Approximate Interpretation

(1) a. There were one hundred people at the rally.
b. Jane arrived at three o'clock.
c. The meeting lasted forty-five minutes.
d. The rope is fifty meters long.

(2) a. There were about one hundred people at the rally
b. Jane arrived at approximately three o'clock.
c. The meeting lasted roughly forty-five minutes.
d. The rope is exactly fifty meters long.

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Agenda

- Two theories of imprecision
- Evidence for scale granularity
- Unexplained data
- The Ruler Model (work in progress)
- Consequences and extensions

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Pragmatic Halos
(Lasersohn 1999; Lauer 2012)

- Imprecision ⇔ not true, but close enough to the truth
- In addition to its denotation, each expression of the language is associated with a pragmatic halo:
  - Entities of same semantic type as denotation, differing from it in only pragmatically ignorable ways

    $[\![3\text{ o'clock}]\!] = 3{:}00$

    $H_C(3{:}00) = \{\ldots i, j, 3{:}00, k, l, \ldots\}$
- Halos derived compositionally
- Approximators operate on pragmatic halos:
  - *Exactly*, etc: shrink halo
  - *Roughly, approximately*, etc: expand denotation to encompass halo

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Scale Granularity
(Krifka 2007, 2009, Sauerland & Stateva 2007)

- Both can be true – or at least felicitous

  It is 600 km from Berlin to Rotterdam.
  It is 611 km from Berlin to Rotterdam.
- **Explanation**: The results of measurement can be reported w.r.t scales that vary in their level of granularity, i.e. their density of representation points
  - 1km-2km-3km-4km- …
  - 10km-20km-30km-40km-50km-…
  - 50km-100km-150km-200km-…
  - etc.
- Approximate ⇔ coarse grained scale; exact ⇔ fine grained scale
- Approximators (e.g. *roughly, exactly*) set granularity level

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Fact 1: Round⇔imprecise

- Tend to be interpreted approximately:
  - (1) a. There were one hundred people at the rally. **100**
  - b. Jane arrived at three o'clock. **3:00**
  - c. The meeting lasted forty-five minutes. **45**
  - d. The rope is fifty meters long. **50**
- Necessarily interpreted precisely:
  - (3) a. There were ninety nine people at the rally. **99**
  - b. Jane arrived at three-oh-one. **3:01**
  - c. The meeting lasted forty-three minutes. **43**
  - d. The cable is fifty-one meters long. **51**

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Fact 2: Roundness is gradable

- Both 90 and 100 round, but 100 is intuitively rounder, and allows more approximate interpretation
  - (4) a. There were one hundred people at the rally.
  - b. There were ninety people at the rally.
    - Also e.g. 3:00 vs. 3:05 vs. 3:01
- Unpublished data from Cummins et al. (2012):
  - About150: 140-160 About 130: 125-135
- Jansen & Pollmann (2001): Roundness defined in terms of divisibility properties
  - Single digit multiple of 10^n, $2x10^n$, $5x10^n$, $2.5x10^n$

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Fact 3: 'Roundness' is domain specific

- Per J&P, 50 rounder than 45. But:
  - (5) a. The meeting lasted forty five minutes. **Approx.**
  - b. The meeting lasted fifty minutes. **Precise**
- More examples (based on Krifka 2007):
  - (6) a. I wrote this article in twenty-four hours. **Approx.**
  - b. I wrote this article in twenty-five hours. **Precise**
  - (7) a. The wheel turned one hundred eighty degrees. **Approx.**
  - b. The wheel turned two hundred degrees. **Precise**
    - ➢ Salient higher-order measurement unit

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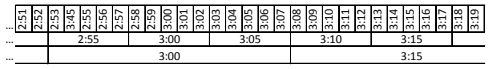
Comparison of Theories - 1

Pragmatic halos:

- "Different in pragmatically ignorable respects" symmetric
  - If 3:01 is in the halo of 3:00, then 3:00 should likewise be in the halo of 3:01
  - No explanation for why some values interpreted more imprecisely than others – or for why this is graded and domain specific

Scale granularity:

- Facts derive from structure of measurement scales
  - E.g. 3:00 occurs on a coarser grained scale than 3:01



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Fact 4: Comparatives precise

- (a) allows approximate interpretation; (b) seems to establish a sharp lower bound
  - (8) a. There were one hundred people at the rally.
  - b. There were more than one hundred people at the rally.
    - ➢ No (??) interpretation on which it is false unless >> 100 attended
    - ➢ No (!) interpretation on which it is true if 99 attended
  - (9) a. The meeting lasted more than forty five minutes.
  - b. The rope is more than fifty meters long.
    - ➢ Intuitions less clear here

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Comparison of Theories - 2

Pragmatic halos:

- Halo of complex expression derived compositionally from halos of constituents
- Would predict that halo of bare numerical expression (e.g.*100*) will be passed up to comparative (e.g. *more than 100*)

Scale granularity:

- Also no immediate explanation – but can seek reasons that comparative selects for fine scale structure

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Fact 5: Approximators are NPIs?

- Apparent support that comparative incompatible with variation in precision level

(10) a. *There were more than roughly one hundred people at the rally.
b. *The meeting lasted more than about forty five minutes.
c. *The rope is more than exactly fifty meters long.

- But…

(11) a. The meeting didn't last more than about forty five minutes.
b. If the meeting lasts more than about forty five minutes…
c. Every meeting that lasts more than about forty five minutes…

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Comparison of Theories - 3

Pragmatic halos:

$[\![\text{roughly } 100]\!] = H_C(100) = \{\ldots, 100, \ldots\}$

- Why restricted to negative downward entailing contexts?
- Type mismatch?
- Interpretation wrong: *no more than roughly100 attended* ≠ there is no number *n* close to 100 such that more than *n* attended

Scale granularity:

- No immediate explanation for contrast – but interpretation correct
  - *no more than roughly100 attended* ≠ 'no more than 100_{coarse} attended'

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Approximating Number Pairs

- Independent evidence for granularity:

There were _____ people in the hall.

2 or 3	*3 or 5
10 or 15	*10 or 13
18 or 20	*18 or 21
30 or 40	*15 or 25
60 or 80	*40 or 80

Rules: - Gap a divisor of both numbers
- Gap a favored number

$$\left\{ \begin{matrix} 1 \\ 2 \\ 5 \\ 2.5 \end{matrix} \right\} \times 10^{n}$$

Pollmann & Jansen 1996; Eriksson et al. 2010

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Summary

- Evidence points to scale granularity as mechanism to account for various aspects of imprecision
- Analysis of imprecision as semantic rather than pragmatic
- Lasersohn's argument against semantic treatment:

(12) Although Mary arrived at three o'clock, she didn't arrive until slightly after three o'clock

- Contradictory feel – but why?
  - Semantic interpretation precise?
  - Granularity reanalysis?
  - Sentence structure (*although*)?

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Fact 5: Endpoints precise

- Clearest with overt approximators:

(13) a. Roughly 60% of our students are from New York State.
b. ??Roughly 100% of our students are from New York State.
c. ??Roughly 0% of our students are from New York State.

➢ Competition with *almost zero/absolutely 100%*, etc?

- Also with bare measure expressions:

(14) a. 60% of our students are from New York State.
b. 100% of our students are from New York State.
c. 0% of our students are from New York State

cf. *'Sales rose by (roughly) one hundred percent'*

➢ Not explained by existing models of granularity

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Fact 6: Granularity at two levels

- Granularity-based scalar implicatures (Cummins et al. 2012):

(15) a. More than 100 people attended the meeting about the new highway construction project..
↝ No more than 150 attended

b. More than 110 people attended the meeting about the new highway construction project.
↝ No more than 120 attended

➢ Semantic meaning involves fine-grained scale
➢ Pragmatic inferences calculated w.r.t. coarser-grained scale

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Objectives

Formal model of granularity that:

- Accounts for both covert imprecision (a) and overt precision regulation (b):
  a. There were one hundred people at the rally.
  b. There were about/roughly/approximately/exactly one hundred people at the rally
- Is integrated within a more general model of scalarity / scale structure
- Explains other relevant data (which?)

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Ruler Model

- Continuous scale…
- …on which a discrete structure is imposed via markings corresponding to conventional measurement units…
- …that are organized into a nested, hierarchical structure…
- …that provides the basis for measurement at varying levels of precision.

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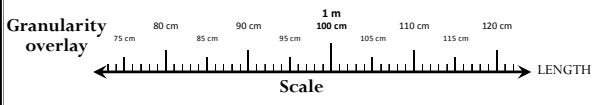
Structure of Granularity



- A **scale** is a triple $S=\langle D, >, DIM\rangle$, where
  - D is a set of degrees
  - $>$ is an ordering relation on D
  - DIM is a dimension of measurement
    - Assumptions: D is dense (except possibly for DIM=cardinality), $>$ is a linear order

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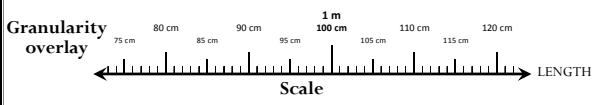
Structure of Granularity



- A **granularity overlay** *Gran(S)* for a scale *S* is a set of sets of degree names, organized into a hierarchical, nested structure
  - Each level of the hierarchy consists of degrees that represent alternatives to one another

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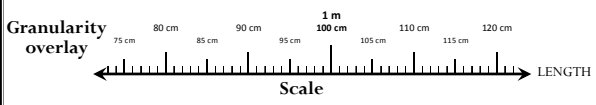
Structure of Granularity



$Gran(S_{LENGTH})$ =

…
∩
{0, 100 cm, 200 cm, …}
∩
{…50 cm,100 cm, 150 cm, …}
∩
{…80 cm, 90 cm, 100 cm, 110 cm, 120 cm, …}
∩
{…90 cm, 95 cm, 100 cm, 105 cm, 110 cm, …}
∩
{…, 97cm, 98cm, 99cm, 100 cm, 101 cm, 102 cm, …}
∩
…

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Granularity Overlay

- Typical levels based on:
  - Powers of 10: {1,2,3,…}, {10, 20, 30, …}, etc.
  - Results of halving: {5, 10, 15, …}, etc.
  - Results of doubling: {2, 4, 6, …}, etc.
- Other domain-specific options:

$Gran(S_{DURATION})$ =

…
∩
{…60 min, 120 min, …}
∩
{…15 min, 30 min, 45 min, 60 min, …}
∩
{…35 min, 40 min, 45 min, 50 min, 55 min, …}
∩
{…, 43 min, 44 min, 45 min, 46 min, 47 min …}
∩
…

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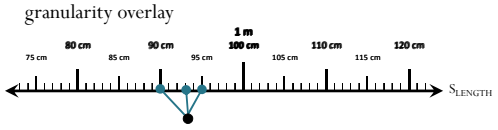
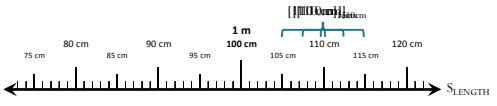
Imprecision – First Attempt

- Measurement always reported w.r.t. some level of the granularity overlay



- Measure expressions always interpreted w.r.t. some level of the granularity overlay

$[\![\text{one hundred ten cm}]\!]^g = [110\text{ cm}]_{gran}$



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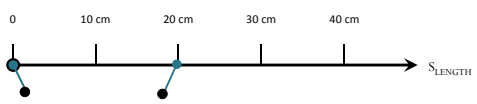
A problem with endpoints

- What is wrong with this?



- *Zero* cannot be interpreted as 'roughly 0'

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Structure of Granularity

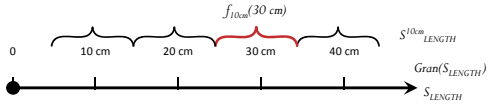
- Granularity functions f_{gran} (Sauerland & Stateva 2011) map points to intervals centered on them

(15) $[\![30\text{ cm}]\!]^g = f_{gran}(30\text{ cm}) = (30\text{ cm} - gran/2,\ 30\text{ cm} + gran/2)$

(16) a. $f_{1cm}(30\text{ cm}) = (29.5\text{ cm},\ 30.5\text{ cm})$

b. $f_{10cm}(30\text{ cm}) = (25\text{ cm},\ 35\text{ cm})$

- Each layer of the *Gran(S)* and its associated granularity function f_{gran} generates a derived scale S^{gran}



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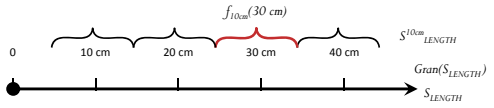
Structure of Granularity

- All interpretation relative to derived scale
- Approximators determine selection of f_{gran}
- Only trivial granularity function f_0 can apply to scalar endpoints – hence endpoints are interpreted precisely
  - **Claim:** this reflects a basic property of imprecision, namely that it is symmetric (cf. Lasersohn 1999, Dehaene 1992)



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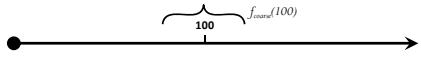
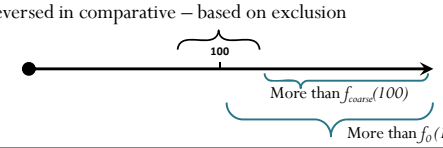
Precise interpretation of comparatives

- Krifka (2007, 2009): Principles of strategic communication → measure expressions tend to be interpreted w.r.t. coarsest granularity level at which they occur
  - If an expression is ambiguous between two meanings, one of which is more likely, hearer can assume that meaning (Parikh 2001)
  - Approx. interpretation more probably than exact interpretation



- Reversed in comparative – based on exclusion



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Implicatures with Comparatives

- Granularity overlay exists independently of coarse-grained derived scales it generates
- Elements of given layer can act as alternatives for purposes of pragmatic inferencing – even when semantic meaning is computed w.r.t. finer-grained scale
  - *More than 100* implicates *not more than 200/150/110*/etc.
  - *More than 110* implicates not more than *110/105*/etc.

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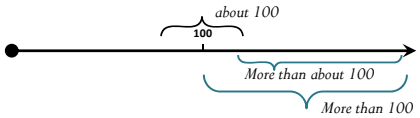
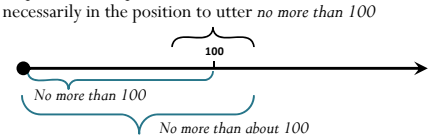
Approximators in Comparatives

- Restriction to negative sentences / NPI contexts pragmatic
  - A speaker in the position to utter *more than about 100* is also in the position to utter *more than 100* – latter favored by brevity



  - A speaker in the position to utter *no more than about 100* is not necessarily in the position to utter *no more than 100*



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Consequences and questions

- Granularity inherently linked to degree names. Does granularity exist in the absence of numerical degrees?
  - Granularity vs. tolerance
- Endpoints necessarily interpreted precisely → consequences for current theories of 'absolute' gradable adjectives such as *clean* and *full*, whose interpretations are based on scalar endpoints

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Also scale granularity?

a full wine glass

an empty theater

- A clean kitchen knife vs. a clean scalpel
- *Completely clean*, *slightly dirty* (Sassoon 2012)
  - On the present analysis: NO. But what then?

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Thank you!

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