Interpreting by the gap hypothesis and pragmatic reasoninge

A major problem for any theory of vagueness is posed by the Sorites paradox. According to the radical pragmatic solution to the problem (Gaifman, 2010; Pagin, 2011; Rayo ,2010, and van Rooij, 2010, arguably all based on Wittgenstein, 1953), we can appropriately use a predicate P in a context if and only if it helps to clearly demarcate the set of objects that have property P from those that do not (the *gap-hypothesis*). This solution seems natural: the division of the set of all relevant objects into those that do have property P and those that do not is (i) easy to make and (ii) worth making. In those circumstances, the tolerance principle $(\forall x, y((Px \wedge x \sim_P y) \rightarrow Py))$ will not give rise to inconsistency, but serves its purpose quite well. Only in exceptional situations i.e., when we are confronted with long sequences of pairwise indistinguishable objects — do things go wrong.

Unfortunately, there is a major problem with this apporach: the gap hypothesis seems too strong: Even if there is no clear demarcation between the bigger and the smaller persons of the domain, certainly the tallest person can be called 'tall'. Thus, the gap-hypothesis as usually implemented doesn't allow for exceptional situations.

We will implement the gap-hypothesis such that it *can* also account for the exceptional situations, by combining Cobreros et al's (2012) three-valued semantic analysis of vagueness with a non-monotonic approach to account for the distinction between normal and exceptional circumstances. We will assume that sentences will be interpreted as true as possible, and (following Cobreros et al, to appear), we will account for this defining a pragmatic interpretation-relation, $PRAG$, which in turn is defined in terms of van Fraassen's (1969) fine-grained notion of meaning (truth-makers). In terms of $PRAG$, we will define a non-monotonic consequence relation, $\models^{prpr}$, that interprets premises and conclusion as strongly as possible. In contrast to our earlier papers we will propose that there is a difference between similarity-statements that are strictly true, and those that are only tolerantly true.

It will turn out that with these two different notions of truth similarity statements can have (strict is better), inferences 1 and 2 are predicted to be valid,[1] but 3 is not:

(1) Tolerance, $Px_1, \neg Px_n \models^{prpr} \neg(x_1 \sim_P x_2 \wedge \cdots \wedge x_{n-1} \sim_P x_n)$

(2) (Tolerance,) $Px_1, x_1 \sim_P x_2 \cdots x_{n-1} \sim_P x_n \models^{prpr} Px_n$

(3) Tolerance, $Px_1, \neg Px_n, x_1 \sim_P x_2 \wedge \cdots \wedge x_{n-1} \sim_P x_n \not\models^{prpr} Px_n$

Prediction 1 basically says that if we have a sequence going from truth value 1 to 0, you expect this to be due to a gap (a pair x_i, x_j such that $x_i \not\sim_P x_j$). Prediction 2 means that if you explicitly say that $x \sim_P y$ (and do not say much more) then you expect that Px and Py have the same truth value: in this case, both value 1. Prediction 3 shows that this expectation can be *cancelled* if it is explicitly said that another individual in the transitive closure of the similarity relation (of course, the similarity relation is only transitively closed with respect to strict truth) does not have property P. This shows that $\models^{prpr}$ is non-monotonic.

Making use of the new consequence relation $\models^{prpr}$ we can diagnose the Sorites reasoning with the tolerance principle as explicit premise as invalid, even though all the premises are true and all the steps are valid. We will discuss whether and how our new approach can account for the intuition that if we express that John is borderline tall by the contradiction $Tj \wedge \neg Tj$, conjunction-elimination should not be valid $Tj \wedge \neg Tj \not\models^{prpr} Tj$.

[1] Though not if one knows that $x_1, \cdots x_n$ are all the individuals.

References

[1] Alxatib, S., P. Pagin and U. Sauerland (2013), 'Acceptable contradictions: pragmatics or semantics? A reply to Cobreros et al., *Journal of Philosophical Logic*, **42**: 619-634.

[2] Cobreros, P., P. Egré, D. Ripley, and R. van Rooij, (2012), 'Tolerant, classical, strict', *Journal of Philosophical Logic*, **41**: 347-385.

[3] Cobreros, P., P. Egré, D. Ripley, and R. van Rooij, (to appear), 'Pragmatic interpretations of vague expressions', *Journal of Philosophical Logic*.

[4] Fraassen, B. van (1969), 'Facts and tautological entailments', *Journal of Philosophy*, **66**: 477-487.

[5] Gaifman, H. (2010), 'Vagueness, tolerance and contextual logic', *Synthese*, **174**: 5-46.

[6] Pagin, P. (2011), 'Vagueness and domain restriction', in P. Egrë and N. Klinedinst (eds.), *Vagueness and Language Use*, Palgrave MacMillan, Basingstroke, pp. 283-307.

[7] Rayo, A. (2010), 'A metasemantic account of vagueness', in R. Dietz and S. Moruzzi (eds.), *Cuts and Clouds: Vagueness, its Nature and its Logic*, Oxford University Press, Oxford, pp. 23-45.

[8] Rooij, R. van (2010), 'Vagueness and linguistics', in Ronzitti (ed.), *Vagueness: A Guide*, 2010, Springer, pp. 122-170.

[9] Wittgenstein, L. (1953), *Philosophische Untersuchungen/Philosophical Investigations*, translated by G.E.M. Anscombe, Blackwell, Oxford.