Aristotelian and Duality Relations with Proportional Quantifiers

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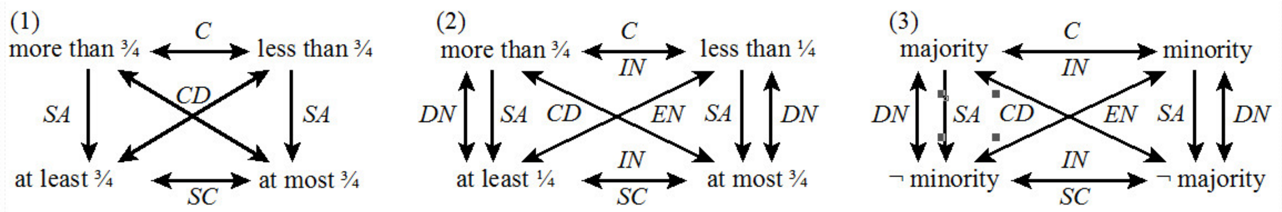
The *Aristotelian* relations of contradiction (CD), contrariety (C), subcontrariety (SC) and subalternation (SA) have been argued to be conceptually independent of the *duality* relations of internal negation (IN), external negation (EN) and dual negation (DN) [1, 2, 4, 5]. For any fragment of 4 formulas (from a logical language $\mathcal{L}$ for a logical system S) which is closed under negation — i.e. which consists of two pairs of contradictories — the former set of relations can be diagrammatically represented as a (possibly degenerate) *Aristotelian square*, whereas the latter set gives rise to a (possibly degenerate) *duality square*.

The central aim of the presentation is to chart which of the above logical relations hold between quantificational formulas expressing the notion of *proportionality*. Two types of expressions will be distinguished: (i) *explicit proportionals* such as *at least two thirds of the A's are B* or *less than 20 percent of the A's are B*, in which the proportion is explicitly referred to in terms of fractions or percentages; and (ii) *implicit proportionals* such as *a minority/majority of the A's are B*, in which the actual proportion remains implicit.



Explicit proportionals will be argued to give rise to (at least) two constellations: (i) the square in Figure 1 is an Aristotelian square only, wheras (ii) the square in Figure 2 is both an Aristotelian and a duality square. Implicit proportionals, then, automatically yield 'double' squares, as in Figure 3. Finally, since these proportional expressions are generalised quantifiers, their monotonicity properties will also be studied [3].

References

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