PACKAGING COMPARATIVE THOUGHTS

LEUVEN WORKSHOP ON DEGREE SEMANTICS | ALEXIS WELLWOOD

Comparative thoughts

What shape do thoughts about something's being greater or equal to another take? What are they about? These questions should be illuminated by typological variation in the linguistic realization of comparatives. Ideally, analyses will keep something like the No Containment Condition in mind:

> No Containment Condition (NCC): No head's semantic representation can contain another's. [Dunbar & Wellwood 2016, 10]
>
> Containment: v_1 is contained within v_3 if there is some composition rule $q \in Q$, Q the set of possible interpretation rules, and some $v_2 \in D$, D the set of possible interpretations of individual heads, such that $q(v_1, v_2) = v_3$.

Language builds recipes for the construction of thoughts (Pietroski 2010), themselves bearers of 'aboutness' properties (Chomsky 2000); the *how* is best revealed through study of fine-grained logical form (Hunter & Wellwood forthcoming).[1] Model theory then connects thought with world.

I aim to characterize a series of derivationally-related thoughts, with truth conditions like:

	states (≈ADJ)	**individuals** (≈N)	**events** (≈V)
lexical ordering	$s_1 \succ_A s_2$	$x_1 \succ_N x_2$	$e_1 \succ_V e_2$
pronunciation	'exceeds'	'superpart'	'superpart'
equivalence ordering	$\bar{s_1}^A \succ_{E^A} \bar{s_2}^A$	...	...
degree ordering	$\mu(\bar{s_1}^A) >_{\mu^{E^A}} \mu(\bar{s_2}^A)$	...	...

(The appendix begins to work out this neodavidsonian spin on the model theory in Bale 2008.)

The typological picture

Plausibly true?:

> The form of comparatives differs widely across languages. The implicational universals in this space suggest that we should not think of the relevant class of thoughts as relatively small and homogeneous, just realized in various distinct ways. Instead, we should limn the typology with a representational theory built out of a series of elements whose interpretation depends on prior representational choices. [cf. Bobaljik 2012]

See Table 1 next page: A rough grasp on (some of) the relevant picture, from a lowly, basically English-only semanticist. My talk will focus on these parts:

- NON-DEG GAs + POS $\sqsubset$ CMPR – Brief review of Wellwood 2015 + Cariani et al. 2023
- MP EXCEPTION – Building on and departing from Anderson & Morzycki 2015
- DIFF MAX + TWO CRISPS + MANIFOLD – **Three derivationally related classes!**

Date: June 1st, 2023.

[1] My 'first step' of interpretation, then, maps to logical forms, technically abusing the $[\![\circ]\!]$ notation. See Wellwood 2020.

TABLE 1. Aspects of the typological picture, and some desiderata.

generalization	what it looks like	what it suggests to me
CROSSCATEGORY	If a language (like English) has degree morphology, it applies not only to gradable adjectives (GAs), but to mass nouns, atelic VPs, plura(ctiona)ls, ...	The semantics of degree is cross-categorial, and **a single expression polymorphically introduces degrees** (Wellwood 2012, 2015, 2019)
NON-DEG GAS	There are languages with GAs and degree morphology (*-er*, *as*, etc.), and languages with GAs but no degree morphology (Bochnak 2015)	**GAs are not essentially degree-involving**; the degree analysis goes with degree morphology (Wellwood 2012, 2015, 2019)
POS $\sqsubset$ CMPR	If a language has dedicated GA comparative forms, these are morphologically more complex than the GA's positive forms (Bobaljik 2012)	A **GA's comparative form is built out of its positive form**, yet in such a way that neither of these forms entail the other (Cariani et al. 2023)
MP EXCEPTION	Few items allow for measure phrase modification (e.g., *2 feet tall*). Many more allow e.g. *Andre the Giant tall*, *dinner at the Ritz expensive* (Wellwood 2014)	MPs don't diagnose a degree semantics for a GA, they require exceptional treatment (Schwarzschild 2005); **need modification by "equivalence" or "kind"**
DIFF MAX	Some languages with degree morphology lack differentials (e.g., *2 feet taller*). A language showing max evidence for a degree-based semantics has differentials.	**Differential comparatives express the most complex comparative thoughts**. They're made out of the parts of less complex comparative thoughts
TWO CRISPS	Some languages have comparatives supporting crisp judgments, but don't utilize the representational resources of differentials (Deal & Hohaus 2019)	**There is a non-degree–based comparative thought that supports crisp judgments**, without all the expressive resources required by a differential
MANIFOLD	Languages vary in whether/how they use standard clauses: none (Motu; Beck et al. 2010); nominals (Japanese; Sudo 2015); SCs (Slavic; Pancheva 2006); tensed clauses (English)	**The semantic complexity of standard clauses tracks their syntactic complexity**

The main takeaways of the discussion to come:

- Gradable adjectives (GA) express properties of states with an associated 'background structure' (cf. Klein 1980, Cresswell 1976, Bale 2008); the positive form is interpreted directly (Cariani et al. 2023). **The simplest comparative thought expresses how two states are ordered with respect to the background structure**. [the morpheme DEG]
- Because MP but not *Andre the Giant* modification is limited, GA modification should be treated heterogeneously. We need 'state-kinds' (Anderson & Morzycki 2015; i.e., 'intensional equivalence orderings'; cf. 'possible individuals' Schwarzschild 2020). **An intermediate comparative thought expresses how two kinds are ordered**. [the morphemes TAK, JAK; also 'UOP']
- Degree-structures are isomorphic to intensional equivalence orders, but they are mathematical objects that support addition, etc. Invoking degrees depends on first invoking kinds, just like invoking kinds depends on first invoking states. **The most complex comparative thoughts order degrees**, and can support mathematical operations. [the morphemes MUCH and DIFF]

In what follows, I lay out a background that gets us most of the way to these three classes of comparative thoughts, and do my best to explicate the structure and meaning of how they're packaged.

... about STATES

Familiar evidence suggests that GAs introduce something that is causally active, measurable, etc.

(1) Because the soup was hot, the girls waited patiently to eat. [cf. Kawamura 2007]

(2) Because Sue left it to boil, the soup was hot.

(3) Al wanted$_{s1}$ the soup to be hot$_{s2}$ because Sue willed it$_{s1/s2}$ so. [Wellwood 2019]

a. 'Sue's willing it so caused Al's desire for hot soup' *because* > [*want* > *hot*]

b. 'Al desired that Sue's willing it would make the soup hot' *want* > [*because* > *hot*]

(4) Al dances/It's hot in the kitchen. [Fults 2006]

Interpreting GAs as introducing 'measurables' supports a uniform characterization of degree morphology (CROSSCATEGORY). At root, then, where $\sigma(c)$ indicates the context assigned by assignment σ[2], we have:

(5) $[\![\text{tall}_c]\!]^\sigma = \lambda s : s \in \mathit{dom}(\preccurlyeq_h).\mathbf{tall}(s, \sigma(c))$ 'being a state of height that counts as tall in $\sigma(c)$'

In the domain condition, $\mathit{dom}(\circ)$ applies to a background structure and returns its domain; $\preccurlyeq_h$ orders height states; '**tall**' indicates a certain 'threshold property' (Cariani et al. 2023). **The positive form, then, isn't a comparative** (N.B. I usually suppress the domain condition):

(6) Ann is tall. 'Ann is in a state of height that counts as tall in $\sigma(c)$'
LF: $\exists$ Ann$_{[Ho]}$ is tall
logical form: $(\exists s : s \in \mathit{dom}(\preccurlyeq_h))(ho(s, \mathbf{a})\ \&\ \mathbf{tall}(s, \sigma(c)))$

Two more pieces are needed for **other plausible, but non-comparative sentences**. I'll use the first widely—a polymorphic unselective binder, (7), here just restricting ν to types c, k, and d.[3]

(7) $[\![\text{UOP}_\chi\ \text{XP}_\chi]\!]^\sigma = \lambda v_\nu : \mathit{type}(\nu, \chi).[\![\text{XP}]\!]^{\sigma[\chi/v]}$

Second, we need a Kleinian operator, (8) (e.g. Deal & Hohaus 2019).

(8) $[\![\text{COMPARED TO}]\!]^\sigma = [\![\text{KLEIN}]\!]^\sigma = \lambda V'_{\nu t} \lambda V_{\nu t}.(\exists v)(V(v)\ \&\ {\sim}V'(v))$ [+Consistency Postulate]

This is enough to handle English *compared to* and Motu A-not-A constructions. At least at a first pass, both express the logical form in (9). Unlike (6), (9) does not entail any positive GA attributions.

(9) Compared to Bill, Ann is tall. [≈ Motu -POS: Ann is tall, Bill is not]
LF: [[UOP$_c$ $\exists$ ANN$_{[\text{Ho}]}$ TALL$_c$] [KLEIN [UOP$_c$ $\exists$ BILL$_{[\text{Ho}]}$ TALL$_c$]]]
logical form: $(\exists c)((\exists s)(ho(s, \mathbf{a})\ \&\ \mathbf{tall}(s, c))\ \&\ {\sim}(\exists s)(ho(s, \mathbf{b})\ \&\ \mathbf{tall}(s', c)))$

... about KINDS

Felicity with MPs often taken to indicate GAs as degree-involving, but **this phenomenon is limited**

(10) 1m tall / 2in long / 2hr long

(11) * 3cm wide / 2ml full / 45 degrees Celsius warm / $3 expensive

[2] I'm coding anything like a free parameter this way. Then it can be bound UOP in (7) as desired.

[3] I assume the input to an expression can be specifically specified as e.g. type e or $\langle e, t\rangle$, or polymorphically: e.g., ν can range over e and v entities. The scope and limits of polymorphic interpretation should be discussed more.

Another kind of modification is productive, but the modifiers **aren't plausibly degree-involving**

(12) Andre the Giant tall / a meterstick long / sitcom comedy long

(13) middle-aged man wide / wine-glass full / Miami warm / dinner at the Ritz expensive

We can take a page from Anderson & Morzycki 2015, who consider modification patterns like *strangely tall.* Their analysis responds to parallels across degree, manner, and kind modification, e.g. in Polish:

(14) *taki pies* ∼ 'such a dog', 'a dog of that kind'

(15) *tak się zachowywać* ∼ 'behave that way' || *jak się zachowywał?* ∼ 'How did he behave?'

(16) *tak wysocki* ∼ 'that tall' || *jak wysocki jest Clyde?'* ∼ 'How tall is Clyde?'

(17) *Taki pies [jak Floyd] szczekał* ∼ 'Such a dog as Floyd barked' [att. to Citko 2000]

On their account, $[\![jak]\!] = [\![tak]\!]$ (but see their fn.12), and both involve (cross-categorial) relations with kinds. Here's how they understand it, metaphysically, in the case of **kinds of states**:

> "Having a certain height is a state, and states, like ordinary individuals, can be arranged into equivalence classes.... the plurality of states of being [a certain height] varies from one world to another.... We can speak of the plurality of all these states across worlds.... This is a state-kind" (*ibid.*, p804)

Here is (my encoding of) their JAK, (19); so far, my TAK flips JAK, (18), and is not demonstrative

(18) $[\![\text{TAK}]\!]^\sigma = \lambda P_{\nu t}\lambda k.(\exists v)(P(v)\ \&\ of(v,k))$ 'given P, being a kind of P-type thing'

(19) $[\![\text{JAK}]\!]^\sigma = \lambda K_{kt}\lambda v_\nu.(\exists k)(K(k)\ \&\ of(v,k))$ 'given K, being of some K-kind'

Here is how A&M's account of (20) looks on my encoding:[4]

(20) Ann is strangely tall. 'Ann's way of being tall is strange'
LF: $\exists$ $\text{ANN}_{[\text{Ho}]}$ [[STRANGELY JAK] TALL_c]
logical form: $(\exists s)(ho(s,\mathbf{a})\ \&\ \mathbf{tall}(s,\sigma(c))\ \&\ (\exists k)(\mathbf{strange}(k)\ \&\ of(s,k)))$

A straightforward extension gets Andre the Giant modification, too, assuming that modifier is stative:

(21) Ann is Andre the Giant tall. 'Ann is tall like Andre the Giant'
LF: $\exists$ $\text{ANN}_{[\text{Ho}]}$ [[[TAK $\text{ANDRE}_{[\text{Ho}]}$] JAK]] TALL_c]
logical form: $(\exists s)(ho(s,\mathbf{a})\ \&\ \mathbf{tall}(s,\sigma(c))\ \&\ (\exists k)((\exists s')(ho(s',\mathbf{g})\ \&\ of(s',k))\ \&\ of(s,k)))$

ER and AS express (polymorphic) greater or equal to relations, not limited to degrees.[5] If we suppose that **only kinds can be measured**, we can interpret MUCH_μ as introducing measures *of kinds*

(22) $[\![\text{AS}_\delta]\!]^\sigma = \lambda v_\nu.v \succcurlyeq \sigma(\delta)$ 'being as great as the salient thing'

(23) $[\![\text{ER}_\delta]\!]^\sigma = \lambda v_\nu.v \succ \sigma(\delta)$ 'being greater than the salient thing'

(24) $[\![\text{MUCH}]\!]^\sigma = \lambda K_{kt}\lambda k.K(\sigma(\mu)(k))$ 'given K, being such that K applies to your measure'

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[4]Notes on the differences here: A&M interpret GAs as relations between states and individuals, don't deal with +/−POS issues, and they identify their state-kinds with (what theorists have thought of as) degrees.

[5](22) and (23) are the **intransitive comparative** meanings. They can be made 'transitive' using UOP in (7).

... about DEGREES

A&M analyze MPs as predicates of kinds, but I'm supposing they name (or predicate of) degrees, (25)

(25) $[\![\text{five feet}]\!]^\sigma = \mathbf{5ft}$ ‘5 feet’, type d

Crucially, MP modification structures do not imply the positive attribution of the GA to the subject. Cariani et al. 2023) (arguably violating the NCC) have various degree morphemes discard the threshold property, and manipulate the GA's background structure instead.

I'll have the morpheme DEG, (26), do this:[6]).

(26) $[\![\text{DEG}]\!]^\sigma = \lambda P_{\nu t}\lambda v_\nu.bg(P,v)$ ‘given P, being in the background associated with P’

Then, this is handy for (exceptional) MP modification, but it also will come in handy for comparatives. As (27) demonstrates, if MPs are degree-denoting, they require a lot of quiet morphology.[7]

(27) Ann is five feet tall. [-POS]
LF: ∃ ANN$_{[\text{Ho}]}$ [[FIVE-FEET [[[MUCH$_\mu$ AS$_\delta$] JAK] UOP$_\delta$]] [TALL$_c$ DEG]]
logical form: $(\exists s)(ho(s,\mathbf{a})\ \&\ bg(s,\mathbf{tall})\ \&\ (\exists k)(\sigma(\mu)(k) \leq \mathbf{5ft}\ \&\ of(s,k)))$

Still, with these pieces, we can construct three distinct sorts of comparatives. Details are in development, but it is exciting that only one of these sorts supports a plausible semantics for differentials.

Building comparative forms

The two pieces missing so far: first, *than/as*—these, I assume, are underlyingly identical just like Hungarian on the surface. (Larson & Wellwood 2015 derives their forms by downward agreement.) It introduces a polymorphic ι/max operator, so it may be used flexibly with UOP.

(28) $[\![\text{THAN}]\!]^\sigma = \lambda P_{\nu t}.(\iota v_\nu)P(v)$ ‘given P, the unique/maximal P’

Second, DIFF—something that introduces degree addition. ((33) works compositionally, but the NCC...)

(29) $[\![\text{DIFF}]\!]^\sigma = \lambda d'\lambda d''.(\iota d)(d = d' + d'')$ ‘given d and d', their sum d’

As we will see, this takes its complement degree and that described by the *than*-clause, adds them, and fills the standard argument of -ER/AS with it.

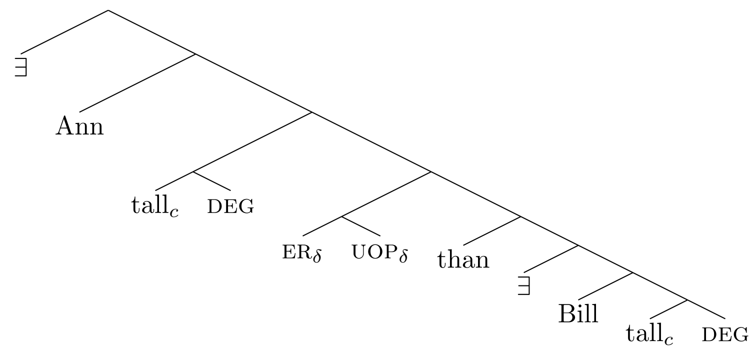
☐ **A comparison of states** (e.g., $s_a \succ_H s_b$)].

Can this work? A comparison of states with an as-local-as-you-like-it standard clause. (Inspired by Sudo 2015's syntactic analysis of Japanese.)[8]

(30) Ann is taller than Bill('s height). ‘Ann is in a height state greater than any Bill is in’
LF: ∃ ANN$_{[\text{Ho}]}$ [[TALL DEG] [[UOP$_\delta$ ER$_\delta$]
[THAN ∃ BILL$_{[\text{Ho}]}$ [TALL DEG]]]]
logical form: $(\exists s)(ho(s,\mathbf{a})\ \&\ bg(s,\mathbf{tall})\ \&\ s \succ$
$(\iota s')(ho(s',\mathbf{b})\ \&\ bg(s',\mathbf{tall})))$

[6]The function bg maps a property P to its background structure O_P, whenever defined. See Wellwood 2019, ‘measurability’.
[7]If they're not degree-involving, they could be interpreted like (20) or (21).
[8]The *than*-clause can attach to AP, or even higher, since it values an implicit argument of ER$_\delta$.

Since the ordering here is just between *actual* states, perhaps these are **non-crisp**?



☐ **A comparison of kinds** (e.g., $\bar{s_a}^H \succ_{E^H} \bar{s_b}^H$).

A still-somewhat impoverished structure. (Inspired by the SC analysis of Pancheva 2006.)

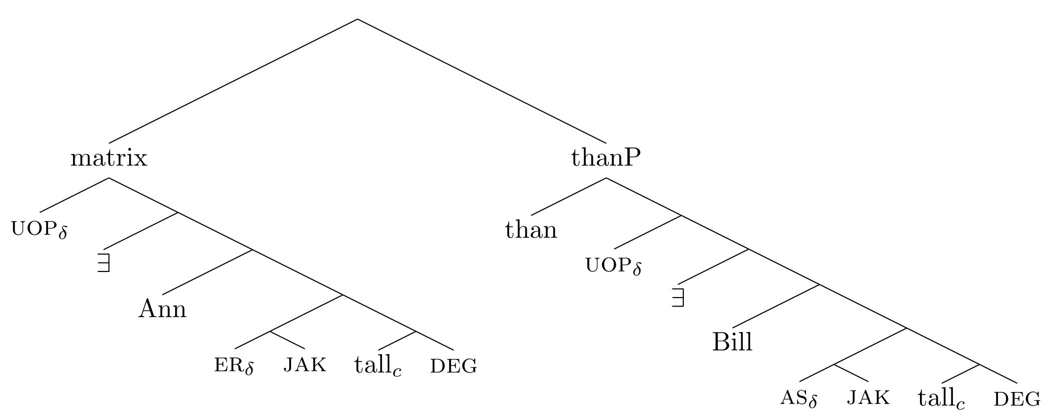
(31) Ann is taller than (be) Bill (tall). ‘Ann’s kind of height-state is greater than any of Bill’s’
<u>LF</u>: [[UOP$_\delta$ ∃ ANN$_{[\text{Ho}]}$ [[ER$_\delta$ JAK] [TALL$_c$ DEG]]]

[THAN UOP$_\delta$ ∃ BILL$_{[\text{Ho}]}$ [[AS$_\delta$ JAK] [TALL$_c$ DEG]]]]

<u>logical form</u>: $(\exists s)(ho(s, \mathbf{a})\ \&\ bg(s, \mathbf{tall})\ \&\ (\exists k)(of(s, k)\ \&\ k \succ$

$(\iota k')(\exists s')(ho(s', \mathbf{b})\ \&\ bg(s', \mathbf{tall})\ \&\ of(s', k')\ \&\ (\exists k'')(k'' \preccurlyeq k'\ \&\ of(s', k'')))))$

Since state-kinds are isomorphic to degrees, these we expect to be **crisp**.



☐ **A comparison of degrees** (e.g., $\mu(\bar{s_a}^H) >_\mu \mu(\bar{s_b}^H)$).

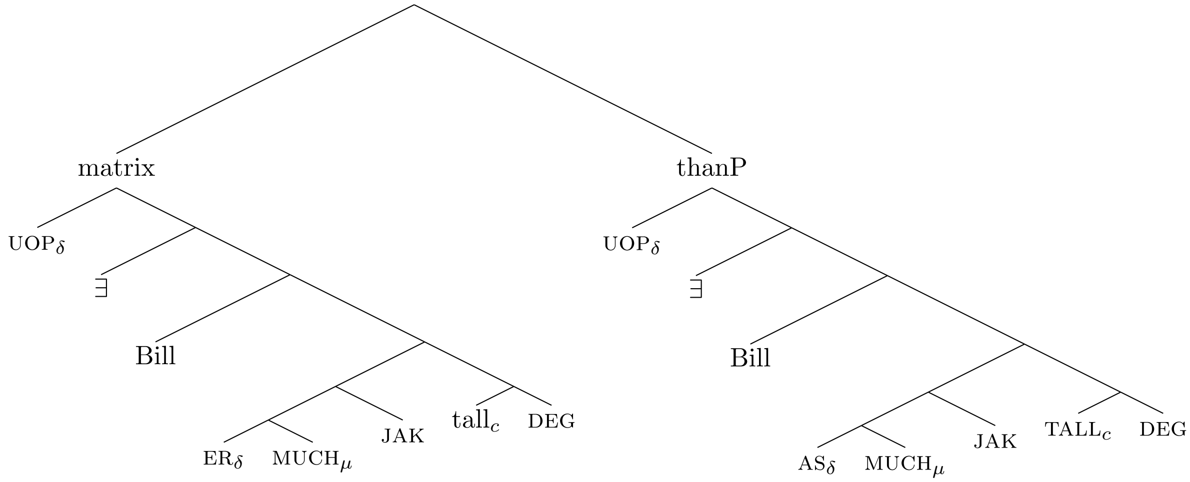
By combining ER and AS with MUCH, UOP can abstract over degrees. These, of course, we expect to be **crisp**. (Finally, the ‘standard’ comparative.)

(32) Ann is taller than (how much) Bill (is tall).
LF: [[UOP$_\delta$ ∃ ANN$_{[\text{Ho}]}$ [[ER$_\delta$ MUCH$_\mu$] JAK] [TALL$_c$ DEG]]

[THAN UOP$_\delta$ ∃ BILL$_{[\text{Ho}]}$ [[AS$_\delta$ MUCH$_\mu$] JAK] [TALL DEG]]]

logical form: $(\exists s)(ho(s, \mathbf{b})\ \&\ bg(s, \mathbf{tall})\ \&\ (\exists k)(of(s, k)\ \&\ \sigma(\mu)(k) >$

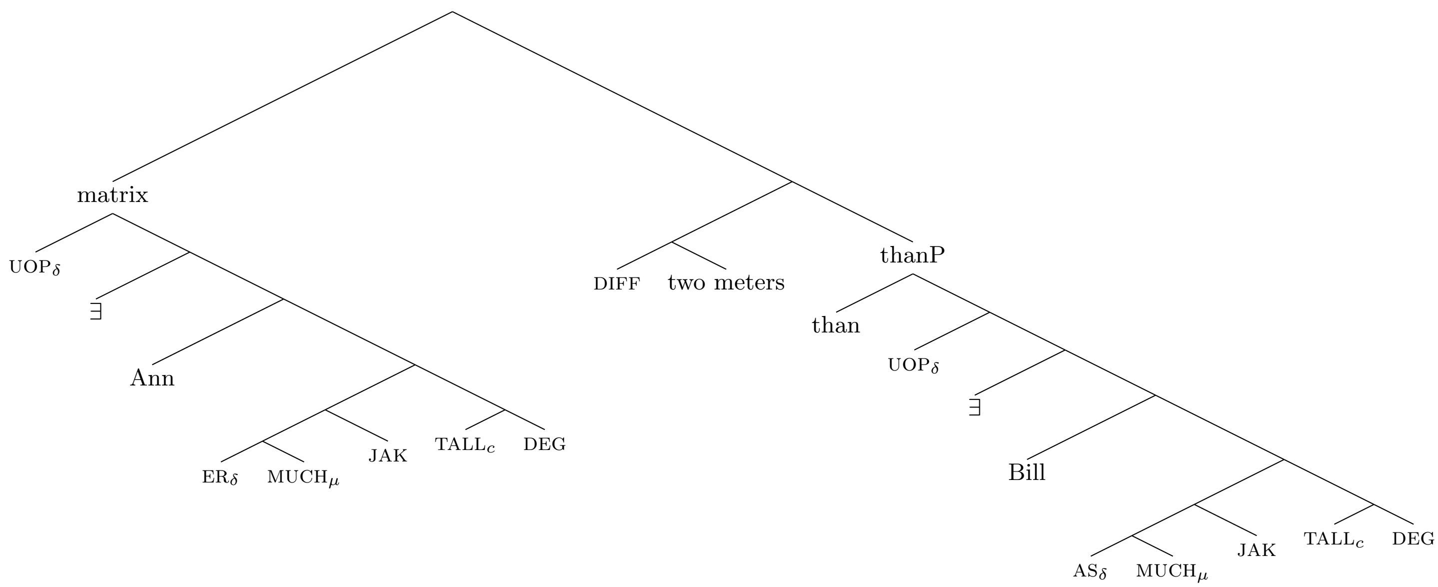
$(\iota d)(\exists s')(bg(s', \mathbf{tall})\ \&\ (\exists k')(of(s', k')\ \&\ \sigma(\mu)(k) \geq d))))$



☐ **... and adding addition** (e.g., $\mu(\bar{s_a}^H) >_\mu \mu(\bar{s_b}^H) + 2m$).

If I had time, I'd try to do this in a way that is more responsive to Schwarzschild 2020 (START and END of relevant degree intervals) and Zhang & Ling 2021 (interval arithmetic)

(33) Ann is two feet taller than Bill. ‘Ann's height state is of a kind measuring greater than...’
<u>LF</u>: [UOP$_\delta$ ∃ ANN$_{[\text{Ho}]}$ [[[ER$_\delta$ MUCH$_\mu$] JAK] [TALL$_c$ DEG]]
[[2INCHES DIFF] [THAN UOP$_\delta$ BILL$_{[\text{Ho}]}$ [[AS$_\delta$ MUCH$_\mu$] JAK] [TALL DEG]]]]
<u>logical form</u>: $(\exists s)(ho(s,\mathbf{b})\ \&\ bg(s,\mathbf{tall})\ \&\ (\exists k)(of(s,k)\ \&\ \sigma(\mu)(k) >$
$(\iota d')(d' = \mathbf{2m} + (\iota d)(\exists s')(bg(s',\mathbf{tall})\ \&\ (\exists k')(of(s',k')\ \&\ \sigma(\mu)(k') > d)))))$



Conclusions

- By interpreting GAs as predicates of states, we can explicitly map them to kinds, and in turn to degrees, in the syntax. This provides a flexible representational vocabulary in which to couch typological descriptions.
- More work is needed, of course, to determine the appropriate vocabulary, its theoretical scope and limits, and to ground it in the specific details of the typological picture.

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Appendix: Working on the model theory

I begin to modify some of what Bale 2008 left 'under the hood' for his GA analyses, to adapt them for the kind of picture developed here.

Variable/constant name assumptions:

sort	**type**	**variables**	**constants**
ordinary individuals	e	$x, x', x'', ...$	$a, b, c, ...$
contexts	c	$c, c', c'', ...$	$c_1, c_2, c_3, ...$
states	v	$s, s', s'', ...$	$s_1, s_2, s_3, ...$
kinds	k	$k, k', k'', ...$	$k_1, k_2, k_3, ...$
degrees (i.e., measurement values)	d	$d, d', d'', ...$	$d_1, d_2, d_3, ...$

For Bale (see also Cresswell 1976), the adjectival relation is a **pre-order** (aka **quasi-order**, a transitive, reflexive, and connected binary relation), and its domain is a subset of the domain of ordinary individuals, D_e. Then, some **indifference** relation is used to form equivalence classes (see also Klein 1991). The set of those classes given $\sim$ is a **linear order** (transitive, reflexive, connected, and anti-symmetric).

Aligning the individuals-based and states-based model theory:

	Individual-based (Bale 2008)	**State-based**
base	$\langle D_\zeta, \{\langle x, y\rangle \mid x, y \in D_\zeta$ & x has as much A as $y\}\rangle$	$\langle D_A, \{\langle s_1, s_2\rangle \mid s_1, s_2 \in D_A$ & s_1 is as much A as $s_2\}\rangle$
***A*-indiff.**	$a \sim_\zeta b$ ('a is equivalent to b w.r.t. ζ') iff $(\forall x \in D_\zeta)((a \succcurlyeq_\zeta x \Leftrightarrow b \succcurlyeq_\zeta x)$ & $(x \succcurlyeq_\zeta y \Leftrightarrow x \succcurlyeq_\zeta b))$	$s_1 \sim_A s_2$ ('s_1 is equivalent to s_2 w.r.t. A') iff $(\forall s \in D_A)((s_1 \succcurlyeq_A s \Leftrightarrow s_2 \succcurlyeq_A s)$ & $(s \succcurlyeq_A s_1 \Leftrightarrow s \succcurlyeq_A s_2))$
***A*-equivs**	$\bar{}^{\zeta} : D_\zeta \rightarrow \mathbf{pow}(D_\zeta)$, such that $(\forall x \in D_\zeta)(\bar{x}^\zeta = \{y \mid y \in D_\zeta$ & $x \sim_\zeta y)$	$\bar{}^{A} : D_A \rightarrow \mathbf{pow}(D_A)$, such that $(\forall s \in D_A)(\bar{s}^A = \{s' \mid s' \in D_A$ & $s \sim_A s'\})$
***A*-classes**	$E_\zeta = \{X \subseteq D_\zeta \mid (\exists x \in D_\zeta)(\bar{x}^\zeta = X)\}$	$E_A = \{S \subseteq D_A \mid (\exists s \in D_A)(\bar{s}^\zeta = S)\}$
^E_A-order	$(\forall x, y \in D_\zeta)(\bar{x}^\zeta \succcurlyeq_\zeta \bar{y}^\zeta \Leftrightarrow x \succcurlyeq_\zeta y)$	$(\forall s_1, s_2 \in D_A)(\bar{s_1}^A \succcurlyeq^E_A \bar{s_2}^A \Leftrightarrow s_1 \succcurlyeq_A s_2)$

Some needed differences:

I. Mapping from equivalence orders to degree scales
Bale supposes that degree comparisons involve mapping to a Universal Scale, with degree values calculated from the number of levels in the relevant ^E_A-order. I think that **degree scales Δ are antecedently given**, and that there are structure-preserving maps (i.e., **morphisms**) from A-classes of states to (an element of) Δ

II. Givenness and typology of scales
Bale supposes that 'indirect' comparisons involve A-structures as discussed, but 'direct' involve A-structures whose domain additionally include measurement values. I too think that there are two relevant kinds of base structures; for me the difference consists in whether **possible states** are included in the base (cf. Anderson & Morzycki 2015, Schwarzschild 2020)

III. The structure of scales
The way Bale calculates degrees results in a Universal Scale that is isomorphic to the structure of rational numbers. I assume that degree scales are isomorphic to the structure of real numbers (cf. Fox & Hackl 2006, Gallistel & Gelman 2000).