A family of degrees

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Over-arching question

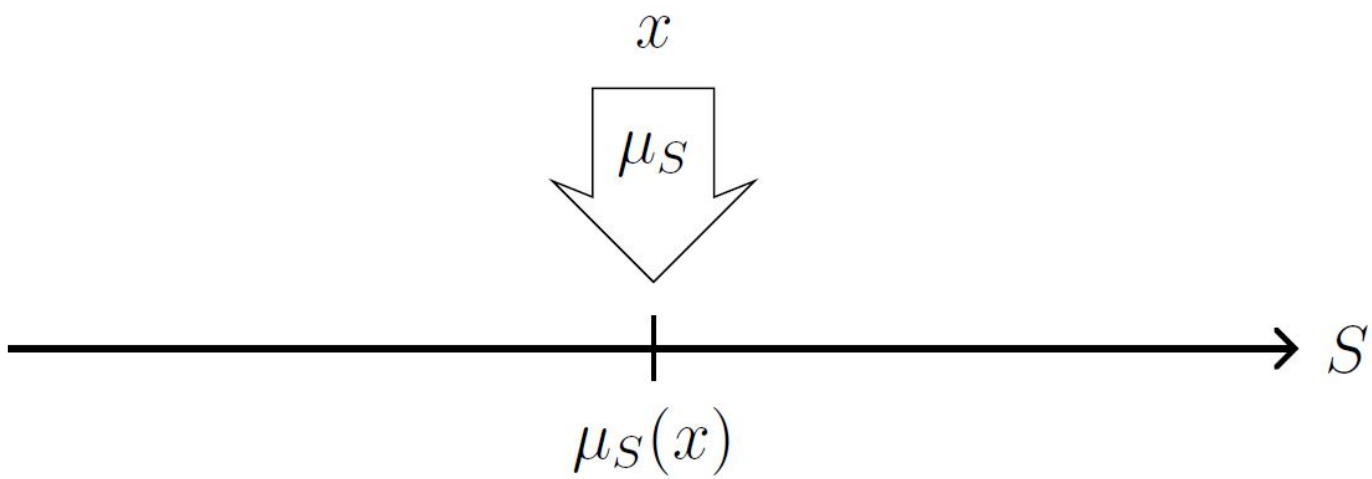
How are our conceptual abilities to determine how much of a certain property an object has represented in the grammar?

A narrower question

What are the adequate (empirically supported) ways of modeling degree expressions? What are the formal properties of those systems?

Degrees, scales and measurement

- A scale $S = \langle D, <, \textsc{dim} \rangle$: a set of degrees, and ordering and a dimension.
- Measure functions μ relate individuals to degrees: μ_S maps some x to the degree d on the scale S that represents x's measure with respect to the dimension DIM:



- No restrictions on what degrees *are*, on the existence of units of measurement, on the properties of $<$, on whether there must a semantic type d, etc.

Two and a half views on degrees

1. The "standard" view: degrees are abstract entities on a scale.

> "whatever they are, they are highly abstract objects"
> [von Stechow 1984, 7]

⇒ Typically, these views take degrees to be *atomic primitives*:

- they exist independent of the entities whose measurements they encode.
- they are assigned their own semantic type, type d, allow quantification and reference.
- they may be points, intervals, directed segments, or complex tuples.

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von Stechow (1984), Heim (2000), Kennedy (2007), Rett (2008), Solt (2009). . .

Two and a half views on degrees

❷ Degrees are derivative, they are produced by comparing individuals.

> “there is no degree without comparison and no comparison without degree”
>
> [Bierwisch 1989, 112]

- They are not primitives, they can be construed as equivalence classes, without the need to posit abstract degrees (Cresswell (1976)): given a relation R (e.g. “taller than”),
  - collect all individuals that are equivalent under R, and
  - define an ordering $\prec$ on the basis of R.
  - They may or may not be assigned a semantic type d.

Klein (1991), Bale (2008), Sassoon (2010), van Rooij (2011), Lassiter (2017). . .

Two and a half views on degrees

- An analogy: degrees as kinds
  - The degree "10kg" consists of the plurality of individuals that weigh precisely 10kg (Cresswell (1976));
  - the weight of individuals varies across worlds, so "10kg" can be a function from a world to the plurality of 10kg-individuals in that world.

  ⇒ This is a Chierchia-style kind.

Anderson and Morzycki (2015), Scontras (2017), Luo and Xie (2018), Zhang (2020)...

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Chierchia-style maps "$^{\cap}$" ($\langle e, t\rangle \to \alpha$) and "$^{\cup}$" ($\alpha \to \langle e, t\rangle$):

- $\lambda s\,.\,{}^{\cup}\textsc{10kg}(s)$ [Anderson and Morzycki (2015)]
- ${}^{\cap}\lambda x\,.\,\exists k[\mu_{kg}(x) = 10 \wedge \pi(k)(x)]$ [Scontras (2017)]
- ${}^{\cap}\lambda x\,.\,|x| = n$ [Rothstein (2017)]

Anderson and Morzycki (2015), Scontras (2017), Luo and Xie (2018), Zhang (2020)...

So. . .

- Degrees can be modeled as nominalizations of quantity uniform properties: we only need sets of individuals.
  - This does not entail that degrees *are* kinds.
  - This does not entail that degrees as primitives do not exist.

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- One degree to rule them all?

Today

- Complex degrees potentially contain more information than their primitive alternatives.
- Complex degrees are able to keep track of what degrees are degrees of.
- ? Can we find any evidence for such additional structure?

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❶ A negative result: some of the evidence in favor of complex degrees fails to provide such support. Evidence from Amount Relatives.

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❓ Can we find any evidence for such additional structure?

❶ A negative result: some of the evidence in favor of complex degrees fails to provide such support. Evidence from Amount Relatives.

❷ A positive result: the comprehensive compositional analysis of constructions such as *four pizzas is enough* (Rett (2018)) supports a side-by-side (dual) view of degrees.

What is an Amount Relative (AR)? Carlson (1977a)

(1) It will take us years to drink the champagne they spilled that evening.

[Heim 1987]

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[Heim 1987]

(2) **Object interpretation**
It will take us years to drink the **particular** champagne they spilled that evening.

(3) **Amount interpretation**
It will take us years to drink the **amount of** champagne they spilled that evening.

The challenge

The OBJECT interpretation of (1) corresponds to a restrictive relative clause:

(4) An intersective interpretation [e.g. Quine 1960, Partee 1973]
$\{x : x \textit{ is champagne}\} \cap \{y : \textit{they spilled } y \textit{ that evening}\}$

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The question

How does an AMOUNT interpretation of a relative clause come about?

The challenge

The consensus

ARs involve degree abstraction at the CP level.

Carlson (1977a), Heim (1987), Grosu and Landman (1998, 2017), von Fintel (1999), McNally (2008), Herdan (2008), Meier (2015), Scontras (2017), a.o.

(5) a. $[_{\text{DP}}$ the champagne $[_{\text{CP}}$ that they spilled that evening$]]$

b. $[\![\text{CP}]\!] = \lambda d$. they spilled d-MUCH champagne that evening

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$\{x : x \textit{ is champagne}\} \cap \{d : \textit{they spilled } d\text{–MUCH } \textit{champagne}\}$???

Introducing degrees: compositional challenges

- If $[\![\text{DP}]\!] \in D_e$

  $[\![\text{DP}]\!] = \lambda y\,.\,champagne(y) \wedge |y| = \textsc{max}\big(\lambda d\,.\,\exists x[champagne(x) \wedge \mu(x) = d \wedge \ldots]\big)$

  $\Rightarrow$ D cannot be interpreted (as a definite determiner).

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  $\Rightarrow$ D cannot be interpreted (as a definite determiner).

- If $[\![\text{DP}]\!] \in D_d$
  $[\![\text{DP}]\!] = \textsc{max}\big(\lambda d\,.\,\exists x[champagne(x) \wedge \textit{they-spilled-last-night}(x) \wedge \mu(x) = d]\big)$
  $\Rightarrow$ We need to figure out composition with the verb.

Properties of ARs

1. *Amount definiteness*:
   ARs refer to a definite amount.
2. *Entity indefiniteness*:
   The head of the relative clause is interpreted as an indefinite.
3. *Identity*:
   ARs require a comparison of two amounts of the **same** stuff.

Definiteness (❶ and ❷)

(6) It would take us years to drink **champagne in that amount**.
[where *that amount = the amount of champagne that they spilled that evening*]

- definite amount: the specific amount of champagne that they spilled that evening.
- indefinite *champagne*: no particular champagne would take us long to drink.

Identity (❸)

ARs, unlike other classifier relative clauses, only relate amounts of the same stuff.

(7) a. It would take us years to drink the **amount of champagne** that you drank (of) **wine**.

b. * It would take us years to drink the **champagne** that you drank (of) **wine**.

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b. * It would take us years to drink the **champagne** that you drank (of) **wine**.

(8) [*I drank two liters of champagne in 3 hours, and you drank two liters of wine in 30 minutes.*]

It took me 3 hours to drink the champagne that you drank in 30 minutes. ⇝ False!

Grosu & Landman (1998, 2017)

- ARs motivated the "structured degrees" view of degrees in Grosu and Landman (1998): degree are composed of a property, a measure, and an individual.

(9) Structured degrees:
For all plural individuals X: $\textsc{degree}_p(X) = \langle |X|, P, X \rangle$

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(9) Structured degrees:
For all plural individuals X: $\textsc{degree}_p(X) = \langle |X|, P, X \rangle$

⇒ Individuals may always be retrieved because they exist internal to degrees themselves.

(10) Shifting from degrees to individuals:
$\textsc{substance}(\text{CP}) = \{x : \langle |x|, P, x \rangle \in (CP)\}$

Grosu & Landman (1998, 2017)

- Since DEGREE$_p(X) \in D_d$, the "amount" interpretation relies on DEGREE,
  - $\Rightarrow$ The sortal mismatch remains.
- Since SUBSTANCE(CP) $\in D_e$, the sortal mismatch is solved. "amount" interpretation.
  - $\Rightarrow$ The amount interpretation is lost.

Scontras (2017)

- AR's are also the target of degrees as nominalized (quantity uniform) properties.

(11) DEGREE = $^{\cap}\lambda x\,.\,\exists k[\mu_f(x) = n \wedge \pi(k)(x)]$
where μ_f is a contextually-specified measure,
n is some number in the range of the measure μ_f,
k is a kind,
and π is a contextually-supplied partitioning instantiation.

- E.g.: $[\![\text{10kg}]\!] =^{\cap} \lambda x\,.\,\exists k[\mu_{kg}(x) = 10 \wedge \pi(k)(x)]$

(12) Asymmetric (directional) intersection:

a. $A_{\langle d,t\rangle} \cap P_{\langle e,t\rangle} = \lambda d\,.\,A(d) \wedge \exists y[P(y) \wedge^{\cup} d(y)]$

b. $P_{\langle e,t\rangle} \cap A_{\langle d,t\rangle} = \lambda x\,.\,P(x) \wedge \exists d[A(d) \wedge^{\cup} d(x)]$

ARs through complex degrees

- Both accounts target amount interpretations of ARs.
- Both accounts rely on complex degrees: degrees that keep track of what they are degrees of.
  - For G&S: degrees carry such information internally.
  - For Scontras: degrees as kinds. Just as kinds are kinds of something, so are degrees degrees of something.

A suspicious similarity

Carlson (1977a,b)

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(13) It would take us years to **find** the champagne that they spilled that evening.

(14) We lost the battle because we lacked the soldiers our enemy had.
⇝ the amount of soldiers that our enemy had.
⇝ soldiers as well {trained/motivated/strong} as our enemy's.

A suspicious similarity

(13) It would take us years to **find** the champagne that they spilled that evening.

(15) a. *Kind definiteness*: (13) seems to refer to a definite kind.

b. *Entity indefiniteness*: The head of the RC is interpreted as an indefinite.

c. *Identity*: (13) is about kinds of the **same** stuff.

No subdeletion

Sub-deletion is considered a hallmark of degree abstraction. E.g. comparatives and equatives all allow sub-deletion.

(16) a. I brought more bananas than you brought apples.

b. I brought as many bananas as you brought apples.

c. I brought the amount of bananas that you brought of apples.

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ARs never allow sub-deletion.

(17) a. It will take years to drink the **amount of** champagne that they spilled of wine.

b. * It will take years to drink the champagne that they spilled wine.

No island sensitivity

Degree and negative operators interact (e.g. Rullmann 1995):

(18) a. $\neg \gg \textsc{max}$

b. $^{*}\textsc{max} \gg \neg$

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(19) a. * How many soldiers doesn't the enemy have?

b. * We have more soldiers than the enemy doesn't have.

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(20) a. We won the battle because we had the soldiers that our enemy didn't have.

b. Our school got the fellowship because we had the students that yours didn't have.

No island sensitivity

(21) a. *Tenseless wh-islands*
We won the battle because we had the soldiers that you wondered whether to hire to fight the enemy.

b. *Factives*
We can easily drink the wine that John regretted that he spilled at the party.

c. *Response stance verbs*
We drank the wine that John denied that he spilled at the party.

No relative clause

(22) Amount/Kind interpretations with PPs

a. We lost the battle because we didn't have the soldiers of the Imperial Army.

b. We used to organize a soccer team, but we don't have the students in the department anymore.

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(23) Amount/Kind interpretations with bare DPs

a. We lost the battle because we didn't have the soldiers.

b. We used to organize a soccer team, but we don't have the students anymore.

Recap

- Data corroborates the *a* parallelism between Kind and Amount interpretations.
- Arguments against a degree-based analysis of Amount interpretations.
  - There is no evidence of degree abstraction in relative clauses, even when they permit an Amount interpretation.
  - Even if there was such evidence, we would still have to explain why Amount interpretations are possible in the absence of RCs altogether.

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⇒ Subsuming AMOUNT interpretations under (some form of) KIND interpretations is not only defensible, but desirable.

Disjointness condition

(24) [*Fido is a watch dog and a border collie. Fido is sitting in the next room.*]

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(26) DISJOINTNESS CONDITION [Carlson 1977b]

A kind-referring expression can only refer to a contextually defined subset of all the possible subkinds that the noun is true of, such that:

i. the subkinds in this subset are disjoint and share no realizations,

ii. the subkinds collectively cover all the space of realizations of the kind.

Degrees as equivalence classes

Take a relation $\succeq_A$, reflective of our conceptual ability to determine, from any two individuals, which has more of a certain quality than another.

(27) $\langle D_{tall},\ \{\langle x, y\rangle\ :\ x, y \in D_{tall}$ and x is as tall as $y\}\rangle$

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With $\succeq_A$ we can define an equivalence relation as follows:

(28) $x \simeq_A y \leftrightarrow x \succeq_A y \wedge y \succeq_A x$

(29) DEG_{tall} **as a partition**

$d_{5.8f}$:	John, Sue Liz
$d_{5.9f}$:	Mary, Al
d_{6f}:	Bill, Helen
$d_{6.1f}$:	Peggy

(30) DEG_{card} **as a partition**:

d_4:	$j \oplus s \oplus l \oplus m$,...
d_3:	$j \oplus s \oplus l, b \oplus h \oplus d$, ...
d_2:	$b \oplus h, m \oplus a, s \oplus l$,...
d_1:	j, s, l, m, a, b, h, p

Connecting the dots – I

Interpreting a kind-referring expression amounts to finding a suitable equivalence relation. Available equivalence relations vary with context.

(31) I like this kind of dog.

a. *Taxonomic* EqR = *be the same breed as*
   Border collie, beagle, pug…

b. *Size* EqR = *be the same size as*
   d-big, …, *d*+*i*-big

c. *Sui generis* EqR = case-by-case
   Dogs that come to greet you when you come back home, dogs that lick your face, dogs that bite your ankles…

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- In order to partition the domain, we must find an equivalence relation.
- Degrees/Amounts can be understood as equivalence classes arising from some equivalence relation.

$\Rightarrow$ When searching for equivalence relations, nothing precludes us from picking one that delivers amounts.

Illustration

(1) It would take us years to find the champagne that we spilled last night.

It could be that the champagne was of a particular kind:

(32) **Champagne partitioned by taxonomic kinds**

Prestige cuvée
Blanc de noirs
Blanc de blancs
Rosé Champagne

$\rightsquigarrow$ the champagne that we spilled last night was a *prestige cuvée*.

Illustration

But also that it was extremely sweet:

(33) **Champagne partitioned by sweetness in gr. of sugar per litre**

$d < 6gr$
$6.1 < d < 12$
$12.1 < d < 17$
$17.1 < d < 32$
$32.1 < d < 50$
$50.1 < d < 67$
…

$\rightsquigarrow$ the champagne we spilled last night was d-sweet.

Illustration

Or simply a lot of champagne:

(34) **Champagne partitioned by volume**

$0\text{L} \leq d < 1\text{L}$
$1.1\text{L} < d < 2\text{L}$
$2.1\text{L} < d < 3\text{L}$
$3.1\text{L} < d < 4\text{L}$
$4.1\text{L} < d < 5\text{L}$
…

$\rightsquigarrow$ the champagne that we spilled last night was d-much.

Recap

(35) It would take us years to find champagne with some relevant property P of the champagne that we spilled last night

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- The equivalence relation must be one where the individuals in the extension of the modified NP are in a single cell of the partition.
- As long as this requirement is observed, any equivalence relation might do.

$\Rightarrow$ The only difference between Kind and Amount interpretations is that different equivalence relations are picked in different contexts.

Takeway

Conclusion

ARs do not require adding additional structure to our notion of degree because they are **not** degree constructions. Their similarities with Kind interpretations are a red-herring.

❓ Can we find any evidence for such additional structure?

⇒ A negative result: some of the evidence in favor of complex degrees fails to provide such support.

Non-Agreeing Degree Construction

- A “family” of constructions where subjects do not agree with their predicates.

⚠Work in progress! Together with Stephanie Solt.

Non-Agreeing Degree Construction

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(36) **Numeral DPs in Subject position**

a. [Four pizzas] **are** vegetarian. *individual*

b. [Four pizzas] **is** enough. *degree*

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Rett's polysemy

(37) **Numeral DPs in Object position**

a. Jane bought [three pizzas]. **They were** delicious.

b. Jane bought [three pizzas]. **It was** more than we needed.

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a. [Many/Three guests] **are** drunk.

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(39) **Bare plurals**

a. [French fries] **were** eaten by the senators.

b. [French fries] **is** not enough. The senators will need protein.

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(40) **Pseudo-partitives**

a. [Four feet of (the) plywood] **are** warped.

b. [Four feet of (the) plywood] **is** more than Betty asked for.

Rett's polysemy

(41) **Definite DPs**

a. [The paintings he salvaged] **were** damaged.

b. [The paintings he salvaged] **was** enough.

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a. [How many books] **are** on the table?

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(42) **Wh-questions with *how many***

a. [How many books] **are** on the table?

b. [How many books] **is** too many?

(43) **Existential quantifier**

a. [Some (of the) cookies] **are** delicious.

b. [Some (of the) cookies] **is** more than they deserve.

First observations

(44) a. Ten pounds of broccoli are in the refrigerator.

b. Five dogs are in the garden.

⇒ Numerical NPs can be subjects to entity denoting predicates.

First observations

(45) a. Ten pounds of broccoli is too much food.
b. Ten pounds of broccoli is too much weight.
c. Ten pounds of broccoli is too much {work / chopping}.

(46) a. Five dogs is too many pets.
b. Five dogs is too much weight.
c. Five dogs is too much {work / grooming}.

⇒ Numerical NPs can be subjects to "degree" denoting predicates.

First observations

(47) a. Ten pounds of broccoli are in the refrigerator.

b. Ten pounds of broccoli is too much {food/weight/cooking}.

- They allow a singular form of the copula.
- There is lack of existential commitment to any one particular individual.
- NADs are not *about* individuals they are *about* amounts/quantity.

Critical observations

(48) a. Ten pounds of broccoli is {as much weight as I can carry / more food than I can handle / *weight}.

b. Five dogs is not { many pets / more pets than we need / *pets}.

- The critical amount/quantity reading is only available in the presence of a form of *much*/*many* or other degree expression.

Critical observations

(49) a. #These rocks are (not) too much food.
b. #These ideas are (not) too much weight.

(50) a. #10lb of rocks are (not) too much food.
b. #Five ideas are (not) too much weight.

- Subjects must be able to be sensibly predicated of the nominal following *much*, regardless of the form of the subject and the polarity of the sentence.

Critical observations

(51) a. 10lb of broccoli is more weight than 10lb of mushrooms.

b. 10lb of broccoli is more food than 10lb of mushrooms.

- They are scale flexible: the nominal in predicate position determines a scalar dimension.
  - In (51a) 10lb of broccoli is a measure of weight.
  - In (51a) 10lb of broccoli is a measure *related* too food. Nutritional value? Abstract satiating power?

Critical observations

- These critical observations suggest that NAD constructions are degree constructions, where degrees must keep track of what they are degrees of.
- We can provide such structure:
  - A slightly different semantics for *much*.
  - A conception of degrees as nominalized quantity uniform properties.

Base case: individual selecting predicates

- Subject DPs to denote predicates of individuals (portions of broccoli or dog-pluralities.
- The quantity provided by a measure function (μ_{WEIGHT}, $\mu_{\#}$) that maps individuals to *simple* degrees.
- Intersection with the sentential predicate (53) and existential closure.

(52) a. $[\![\text{10lb of broccoli}]\!] = \lambda x.\mathit{broccoli}(x) \wedge \mu_{WEIGHT}(x) = 10lb$

b. $[\![\text{5 dogs}]\!] = \lambda x.\mathit{dogs}(x) \wedge \mu_{\#}(x) = 5$

(53) $[\![\text{in the refrigerator}]\!] = \lambda x.\mathit{in\text{-}the\text{-}refrigerator}(x)$

(54) $\exists x[\mathit{broccoli}(x) \wedge \mu_{WEIGHT}(x) = 10lb \wedge \mathit{in\text{-}the\text{-}refrigerator}(x)]$

Degree-selecting predicates

- Such analysis does not generalize: NAD variants are not *about* broccoli, they predicate a property of an amount/quantity.
- This reading requires *much*.

Degree-selecting predicates

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- This reading requires *much*.
- Our *much* combines with an ordinary predicate P and delivers a set of *complex* degrees.
  - *much* is a "scale builder": it combines with a noun that provides the name of a dimension.
  - it requires its subject to "know" the kind of degree it is, meeting two criteria: (i) that it is quantity uniform and (ii) a subset of the nominal predicate.

(55) $[\![\text{too much}]\!] = \lambda P_{\langle e,t\rangle}.\lambda d_e : \exists X \subseteq P[U_{DIM}(X) \wedge d_e = {}^{\cap}X].\ d > \theta_P$,
where $U_{DIM}(X)$ iff $\forall x, y \in X[x \sim_{DIM} y]$

Degree-selecting predicates

- For *10lb of broccoli is too much food*

(56) $[\![\text{too much food}]\!] = \lambda d_e : \exists X \subseteq [\![\text{food}]\!][U_{DIM}(X) \wedge d_e = {}^{\cap}X]. \, d > \theta_{food}$

(57) $[\![(45a)]\!] = {}^{\cap}(\lambda x.broccoli(x) \wedge \mu_{WEIGHT}(x) = 10lb) > \theta_{food}$

- The subject is a nominalized quantity uniform subset of food-stuff that exceeds a certain threshold "built around" food.

Degree-selecting predicates

- And *I bought too much food*? *This broccoli is too much food*?
- Shift to map a set of complex degrees to the set of entities realizing those degrees:

(58) $[\![\text{SHIFT}]\!] = \lambda D_{\langle d,t\rangle}\lambda x_e.\exists d \in D[x \in^{\cup} d]$

➔ This is possible because complex degrees provide this information for us (cf. Grosu & Landman 1998, 2017).

(59) $[\![\text{This broccoli is too much food}]\!] = \exists d \in D[\textit{this-broccoli} \in {}^{\cup}d]$
where $D = \{d : d > \theta_{food}\}$

More degree-selecting predicates

? *10lb of broccoli is too much weight*

More degree-selecting predicates

❓ *10lb of broccoli is too much weight*

- Plug-and-play:

(60) $[\![\text{too much weight}]\!] =$
$\lambda d_e : \exists X \subseteq [\![\text{weight}]\!][U_{DIM}(X) \wedge d_e = \cap X]. d > \theta_{weight}$

(61) $[\![(45b)]\!] = \cap(\lambda x.broccoli(x) \wedge \mu_{WEIGHT}(x) = 10lb) > \theta_{weight}$

More degree-selecting predicates

10lb of broccoli is too much weight

- Plug-and-play:

(60) $[\![\text{too much weight}]\!] =$
$\lambda d_e : \exists X \sqsubseteq [\![\text{weight}]\!][U_{DIM}(X) \wedge d_e = {}^{\cap}X]. d > \theta_{weight}$

(61) $[\![(45b)]\!] = {}^{\cap}(\lambda x.broccoli(x) \wedge \mu_{WEIGHT}(x) = 10lb) > \theta_{weight}$

- The subject is a nominalized quantity uniform subset of weight-stuff that exceeds a certain threshold "built around" weight.

More degree-selecting predicates

(61) $[\![(45b)]\!] = {}^{\cap}(\lambda x.\textit{broccoli}(x) \wedge \mu_{WEIGHT}(x) = 10lb) > \theta_{weight}$

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- *weight* is an abstract mass noun denoting "dimension-stuff" associated with individuals.
- This can be modeled as the set of weights associated with particular individuals (cf. particularized properties, tropes, modes, accidents. . .).

➔ Numerical NPs like *10lb of broccoli* must have interpretations as nominalization of

  - sets of portions of broccoli
  - sets of weights associated to broccoli

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➔ Numerical NPs like *10lb of broccoli* must have interpretations as nominalization of

  - sets of portions of broccoli
  - sets of weights associated to broccoli

- This is what we gain by allowing *complex* degrees.

Takeway

Conclusion

Certain NAD constructions seem to require additional structure: degrees must track what they are degrees of.

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Certain NAD constructions seem to require additional structure: degrees must track what they are degrees of.

❓ Can we find any evidence for such additional structure?

⇒ A positive result: complex degrees, understood as structures tracking information about their internal composition, are indeed attested.

Some observations

- As presented here, complex degrees *as well as* simple ones allows a compositional analysis of a range of puzzling copular constructions.
  - ⇒ We did **not** say anything about simplex degrees!

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  - ⇒ *10lb of broccoli is 2lb more weight than this sack of potatoes*?

Some observations

- As presented here, complex degrees *as well as* simple ones allows a compositional analysis of a range of puzzling copular constructions.
  - ⇒ We did **not** say anything about simplex degrees!
- What are the formal properties of their scales?
  - ⇒ *10lb of broccoli is 2lb more weight than this sack of potatoes*?
- What is the "right kind" of nominalizing operation? Do we really want to say that *five dogs* is a Kind?

Thank you!

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Basics

(62) **Partition funcion**

A partition $\prod$ is a $\langle kt, kt \rangle$ function such that for any kind K, $\prod(K)$ meets two conditions:

a. $\forall x_o[\exists y_k \in \prod(K)[x_o \leq y_k] \rightarrow \neg\exists z_k \in \prod(K)[y_k \neq z_k \wedge x_o \leq z_k]]$

b. $\forall x_o[x_o \leq K \rightarrow \exists y_k \in \prod(K)[x_o \leq y_k]]$

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b. $\forall x_o[x_o \leq K \rightarrow \exists y_k \in \prod(K)[x_o \leq y_k]]$

(63) $[\![KSK1]\!] = \lambda x_k.\lambda y_k.\, \prod(x_k)(y_k)$

(64) $[\![KSK1]\!]([\![\textsc{dog}]\!]) = \lambda y_k.\, \prod(\textsc{dog})(y_k)$
$= \{\textsc{greyhound}, \textsc{collie}, \textsc{beagle}, \ldots\}$

Ad hoc subkinds

(65) $[\![KSK2]\!] = \lambda x_k.\lambda P_{\langle et\rangle}.\lambda y_k.\, \prod(x_k)(y_k) \wedge {}^{\cap}({}^{\cup}x_k \cap P) \in \prod(x_k)$

(66) $\big[_{\text{DP}}$ the $\big[_{\text{NP1}}\ \big[_{\text{NP2}}\ KSK2$ lions $\big]\ \big[_{\text{CP}}$ that eat people$\big]\big]\big]$.

(67) $[\![\text{NP1}]\!] = \lambda y_k.\, \prod(\textsc{lion})(y_k) \wedge {}^{\cap}(\textsc{lion} \cap [\![CP]\!]) \in \prod(\textsc{lion})$
$= \lambda y_k.\, \prod(\textsc{lion})(y_k) \wedge {}^{\cap}(\lambda x.{}^{*}\mathit{lion}(x) \wedge \mathit{eat\text{-}people}(x)) \in \prod(\textsc{lion})$

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From here on:

- Close with ι, return a definite kind.
- Combine via DKP.

Assessment

(1) It would take us years to drink the champagne that we spilled last night.

(**??**) It would take us years to drink **champagne with some relevant property of the champagne we spilled last night**.

- The desiderata in (**??**) are met: (i) we get a *definite* kind expression (ii) where the head of the relative clause is interpreted as an *indefinite*, and (iii) and the kinds involved are kinds of the same stuff.
- The rampant vagueness (and richness) of these RCs is captured by its high context sensitivity.
- No presence of degree-related effects is expected.
- It is not so surprising that the relative clause is not present.